ECE 20001 | Chpt 1

## Current

$i(t) = \frac{dq}{dt}$ $\qquad q_{total}(t) = \int_{t_0}^{t} i(\tau)\, d\tau$ $\qquad \xrightarrow{i(t)} \; = \; \xleftarrow{-i(t)}$

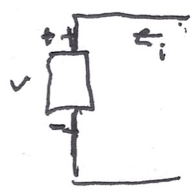
## Voltage

$v_{ab} = v_a - v_b = - v_{ba}$

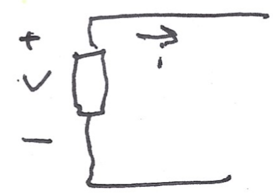
## Power

$P = VI$

Passive sign conv.




active sign conv.

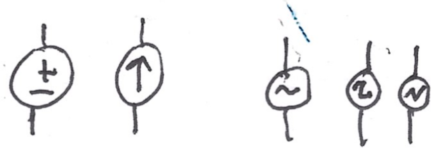



Passive Device (Load)
Consumes power

Active Element (Source)
Delivers Power

## Sources

Independent Sources - Not dep on circuit



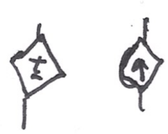
or

Voltage Source

Current Source

AC Sources

Dependent Sources - Dep on Circuit



Voltage Current

VCVS - V controlled V source
CCVS - I contr. V source
VCCS - V contr. I source
CCCS - I contr. I source

## Circuits

Open

Short

Turn off sources
= Short V, Open I

Series

Parallel

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# Chpt 2

## Ohms Law

$$v(t) = i(t)R \quad \text{or} \quad i(t) = \frac{v(t)}{R}$$

## Resistors

$R = \rho \frac{l}{A}$  $\rho$ = resistivity

$C = \frac{1}{R}$ (Conductance)

## Power from R

$$p = vi \Rightarrow p = \frac{v^2}{R} \quad \text{or} \quad p = i^2 R$$

## Kirchoff Laws

KCL: Sum of all currents leaving a node = 0

$$\Sigma i_n(t) = 0$$

Series: $R_{eq} = \Sigma R_N$

Parallel: $R_{eq} = \left(\Sigma \frac{1}{R_N}\right)^{-1}$

KVL: Sum of voltages around a closed path = 0

$$\Sigma v_n(t) = 0$$

## Equivalent Resistance

Turn off indep Sources + Calc Req. (if no dep)

If dep Sources, Use test voltage/current (in test) Calc other value

$$R_{eq} = \frac{V}{I}$$

(Parallel)

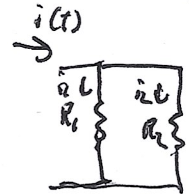
$$i_n(t) = \frac{R_{eq}}{R_n} i(t)$$

aka

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t)$$

or

$$i_2(t) = \frac{R_1}{R_1 + R_2} i(t)$$




(Series)

$$V_n(t) = \frac{R_n}{R_{eq}} v(t)$$

aka

$$v_1(t) = \frac{R_1}{R_1 + R_2} v(t)$$

or

$$v_2(t) = \frac{R_2}{R_1 + R_2} v(t)$$


$$I_n(t) = \frac{R_{eq}}{R_n} i(t)$$

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## Nodal Analysis

1. Number all essential nodes
2. Write KCL eqs for all nodes. Replace any unknown current w/ Nodal voltages. (Ohms Law)
3. $G\vec{V} = \vec{I}$ (Group Eqs into Matrix)
4. Solve using $\vec{V} = G^{-1}\vec{I}$ (G bc most coeffs will be $\frac{1}{R}$ vals)
5. Calc. needed vals from nodal voltages

## Mesh Analysis

1. List all Mesh Currents w/ direction
2. Write KVL eqs for all meshes. Replace any unknown voltage w/ Mesh Currents. (Ohms Law). Remember to write multiple mesh currents if mult. flow through the component
3. $R\vec{I} = \vec{V}$ (Group Eqs into Matrix)
4. Solve using $\vec{I} = R^{-1}\vec{V}$ (R bc most coeffs will be R)
5. Calc needed vals from mesh currents

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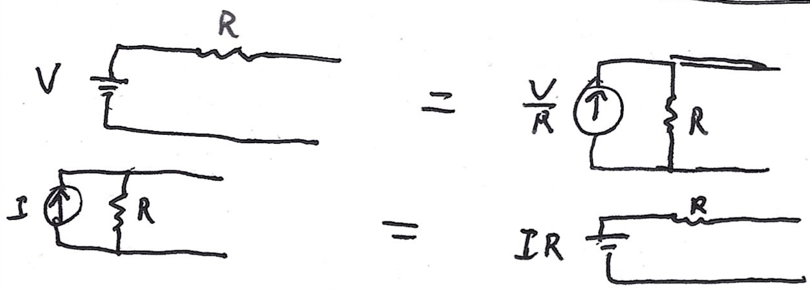
## Basic Principles

Linearity – Scales Linearly

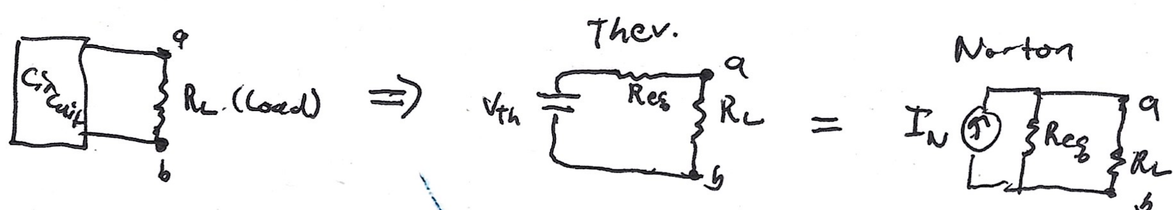
Superposition – Check one indep source at a time, add up effects

## Source Transformation




## Norton / Thevinen

Calc $R_{eq}$, as prev written




$V_{Th} = I_N R_{eq}$

Calc $V_{th}$ by a open circuit (on $R_L$) use KCL/KVL

Calc $I_N$ by a short circuit (on $R_L$)

~~[illegible]~~

## Capacitor

$q = CV$ - based on Voltage $\quad C = \frac{\varepsilon A}{d}$ - based on geometry

$i_c(t) = C\frac{dV_c}{dt}$

$\boxed{\text{Voltage Across a Capacitor is Continuous}}$

- Capacitor behaves as an open circuit in DC stable state circuits and as a short immedietly after connection.

$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(x)\,dx \Rightarrow V_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(x)\,dx + V_c(t_0)$

$P_c(t) = v_c(t)\, i_c(t)$ (Power) $\Rightarrow W_c(t) = \frac{1}{2}CV_c^2(t)$ (Energy)

$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$ Series - Voltage is distr. by ratio of capacitances

$C_{eq} = \sum C_i$ parallel - Voltage across each C is =.

## Inductors

$L = \mu_0 \frac{N^2}{l} S$ - based on geometry

(N turns, l length, S surface Area)

$V_L(t) = L\frac{di_L(t)}{dt}$

$\boxed{\text{Current through an Inductor is Continuous}}$

- Inductor behaves as a short circuit in DC stable state circuits.

$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(x)\,dx \Rightarrow i_L(t) = \frac{1}{L}\int_{t_0}^{t} V_L(x)\,dx + i_L(t_0)$

$P_L(t) = v_L(t)\, i_L(t)$ (power) $\Rightarrow W_L(t) = \frac{1}{2}L i_L^2(t)$

$L_{eq} = \sum L_i$ series

$\frac{1}{L_{eq}} = \sum \frac{1}{L_i}$ parallel

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## RC Circuits

$$v_c(t) = V_{cc} + (V_c(t_0) - V_{cc})e^{-\frac{(t-t_0)}{\tau}} \qquad \tau = RC$$

$t=\tau : 63.2\%$ , $t=3\tau : 95\%$ $t=5\tau : 99.3\%$

$V_{cc} = V_c(\infty)$

## RL Circuits

$$i_L(t) = \frac{V_{cc}}{R} + \left(i_L(0) - \frac{V_{cc}}{R}\right)e^{-\frac{(t-t_0)}{\tau}} \qquad \tau = \frac{L}{R}$$

$t=\tau : 63.2\%$ $t=3\tau : 95\%$ $t=5\tau : 99.3\%$

$i_L(\infty) = \frac{V_{cc}}{R}$

## Basic Sinwave

$$V(t) = V_m \sin(\omega t + \phi)$$

$V_m$ - Amplitude

$\omega$ - radial freq.

$\phi$ - phase shift

$f = \frac{1}{T} = \frac{\omega}{2\pi}$ freq.

$T$ - Time period

## Complex nums

$$X = a + jb \Rightarrow X = r\angle\theta = re^{j\theta}$$

$-\pi < \theta < \pi$

$$r = \sqrt{R(x)^2 + I(x)^2}, \quad \theta = \tan^{-1}\left(\frac{I(x)}{R(x)}\right)$$

$$r = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$X = r_1 \angle \theta_1 \qquad Y = r_2 \angle \theta_2 \Rightarrow$ ~~$XY = r_1 r_2 \angle \theta_1 + \theta_2$~~

$XY = r_1 r_2 \angle \theta_1 + \theta_2 \qquad \frac{X}{Y} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$

$\frac{1}{X} = \frac{1}{r_1} \angle -\theta_1 \qquad X^n = r_1^n \angle n\theta$

$X^* = r_1 \angle -\theta_1 \qquad j = 1 \angle 90°$

## Phasors

$$x(t) = A\cos(\omega t + \theta°) \Rightarrow \tilde{X} = A\angle\theta$$

## Impedence

$\tilde{V} = Z\tilde{I}$

Admittance $\Rightarrow Y = \frac{1}{Z}$

$Z_R = R \qquad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} \qquad Z_L = j\omega L$

# Power

$p(t) = v(t)\, i(t) \Rightarrow$

Instantaneous Power

$$p(t) = \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

## Average Power

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \Rightarrow P = R\left(\tfrac{1}{2} \tilde{V} \tilde{I}^*\right)$$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$V_{eff} = V_{Rms} = V_m/\sqrt{2}$

$I_{eff} = I_{Rms} = I_m/\sqrt{2}$

$$X_{eff} = X_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} x^2(t)\, dt}$$

## Max abs pow

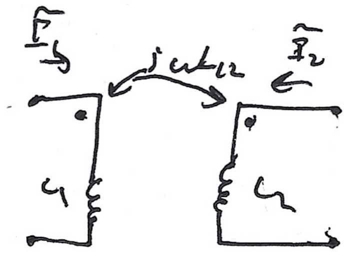
$$P_{Load(max)} = \frac{|V_{th}|^2}{8 R_{eq}}$$

$\Downarrow$ $\Downarrow$

$Z_{Load} = Z^*_{eq}$, $R_{Load} = R_{eq}$, $X_{Load} = -X_{eq}$

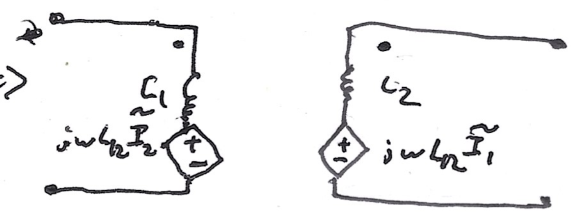
## Inductance

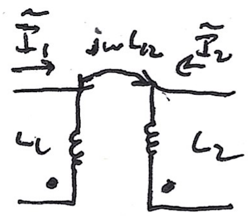
- Self-Inductance - $L = \dfrac{\Phi}{I}$ $\qquad \Phi_{12} = \int_{S_2} \vec{B_1}\, d\vec{S_2}$
- Mutual-Inductance - $L_{12} = \dfrac{N_2 \Phi_{12}}{I_1}$ ($N_2$ = # of turns) (Loop 1 + Loop 2)
- $L_{11} > L_{12}$ bc $\Phi_{11} > \Phi_{12}$ $\Rightarrow$ $L_{12} = L_{21}$
- $V_{M_1} = L_{12} \dfrac{di_2(t)}{dt}$  $\qquad V_{M_1} = -L_{12} \dfrac{di_2(t)}{dt}$ 



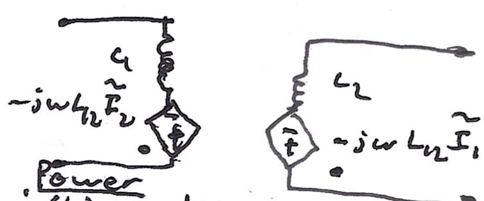

<u>Mutual Inductance</u>

$\Rightarrow$




$\Rightarrow$




Power

$$p(t_1 < t < t_2) = p_2(t) = i_1(t_1) L_{12} \frac{di_2(t)}{dt} + i_2(t) L_2 \frac{di_2(t)}{dt}$$

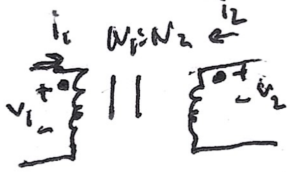
$$p(0 < t < t_1) = p_1(t) = i_1(t) L_1 \frac{di_1(t)}{dt}$$

$$w(t_1 < t < t_2) = L_{12}\, i_1(t_1)\, i_2(t_2) + \tfrac{1}{2} L_2 i_2^2(t_2)$$

$$w_T = \tfrac{1}{2} L_1 i_1^2 + \tfrac{1}{2} L_2 i_2^2 \pm L_{12} i_1 i_2$$ (+ if same side dot)

- Coupling Coeff. $k = \dfrac{L_{12}}{\sqrt{L_1 L_2}}$

<u>Transformer</u>




$\Rightarrow$ $\dfrac{v_2(t)}{v_1(t)} = \dfrac{N_2}{N_1} \Rightarrow n = \dfrac{N_2}{N_1}$

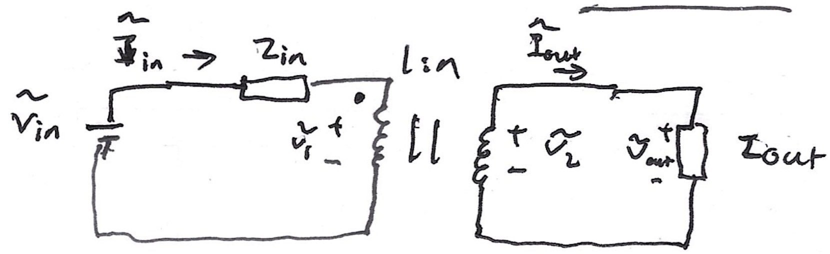
$\dfrac{i_2(t)}{i_1(t)} = -\dfrac{1}{n}$

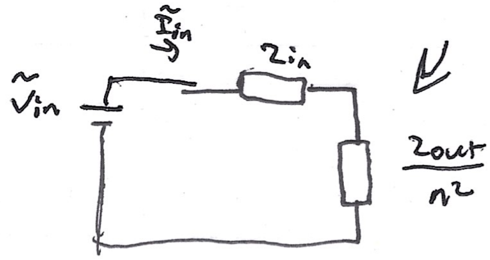
(opp dots)

$\dfrac{v_2(t)}{v_1(t)} = -n$

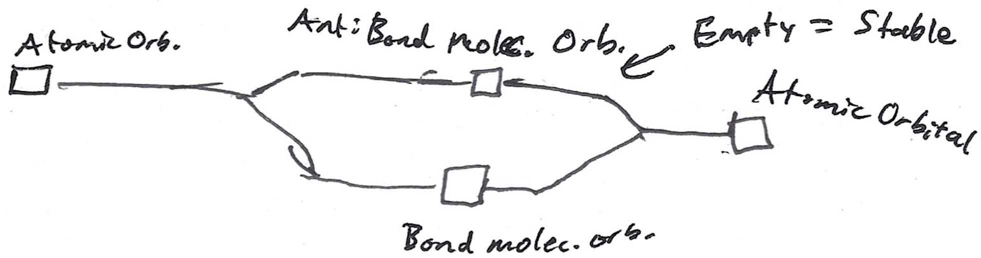
$\dfrac{i_2(t)}{i_1(t)} = \dfrac{1}{n}$

Transformer cont'd

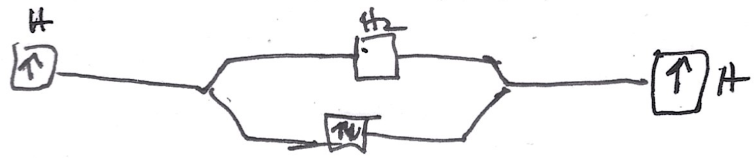


## Molecular Orbitals




$H_2$:




For each orbital one of these exists (p - 3 etc.)

## Silicon Crystal

Anti bond → conduc. band $\qquad$ Si = 4 valence

Bond → valence band.


Band Gap → Ins = big $\quad$ Semi = small $\quad$ Conduc = Overlap

## Doping

(n) Donors $\quad$ N, P, As, Sb, Bi

(p) Acceptors $\quad$ B, Al, Ga, In, Tl

Undoped semi = intrisic semi conductor ⇒ $n = p = n_i$ (intr. carrier density)

n - elec conc. $\qquad n \simeq N_D$ } Doping concentrations

p - hole conc. $\qquad p \simeq N_A$

$np = n_i^2$

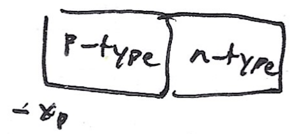
(n) $\quad p \approx \frac{n_i^2}{N_D}$ for $N_D \gg n_i$, $N_D \gg N_A$

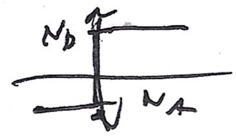
(p) $\quad n \simeq \frac{n_i^2}{N_A}$ for $N_A \gg n_i$ $N_A \gg N_D$

Pn junction = semi trans. from p to n type

## pn junction

Idealized:




$N_D^+ = N_D$ (ionized)

$N_A^- = N_A$

$\rho = q(p - n + N_D - N_A)$

Charge dens.

$$V(f) - V(i) = -\int_i^f \mathcal{E}_x \, dx$$

$$\mathcal{E}_x(x) = \int_{-x_p}^{f} \frac{\rho}{\varepsilon_r \varepsilon_0} \, dx$$

$\varepsilon_r$ = perm. of semi

$\varepsilon_0 = 8.854 \cdot 10^{-12}$

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

## Depletion width

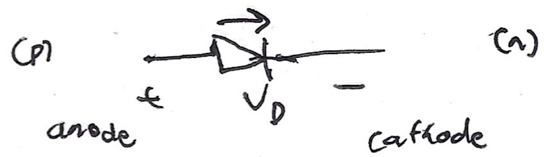
$q N_A V_p = q N_D V_n$ $\qquad V_p = x_p A$, $V_n = x_n A$

$\Downarrow$

$q N_A x_p = q N_D x_n \Rightarrow \qquad x_n = \frac{N_A}{N_D} x_p$

# Diodes

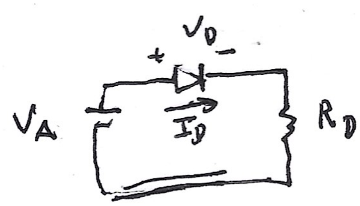
Easy current flow in 1 direction.




$$I = I_0 \left(e^{(qV_A/kT)} - 1\right)$$

q - electron charge
k - boltzman const.
T - temperature
$I_0$ - reverse bias sat. current
$kT/q \approx 0.0259$ (thermal volt. at 300K)

## Simple Diode Circuit




$$\boxed{V_A = V_D + I_D R_D} \Rightarrow V_A = V_D + I_0\left(e^{qV_D/kT} - 1\right) R_D$$

Two methods: Graph/Numeric

Graph: $\frac{V_A - V_D}{R}$ vs $V_D$ and $I_0\left(e^{qV_D/kT} - 1\right)$ vs $V_D$

(x, y) intersection is $(V_{DQ}, I_{DQ})$

Numeric: Calc: $I_D = \frac{V_A - V_D}{R_D}$ (Assume $V_D \approx 0$)

$$V_D = \frac{kT}{q} \ln\left(\frac{I_D}{I_0} + 1\right)$$

Repeat these two until precision reached.

## Simplified Diode Model

$\boxed{i_D = 0,\ v_D < 0 \Rightarrow \text{Diode (reverse bias) is open circuit}}$

$\boxed{i_D > 0,\ v_D \approx 0 \Rightarrow \text{Diode is a short circuit (forward bias)}}$

Guess one & check: (reverse: 0 > V, forward: 0 < I)

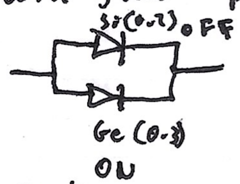
$0.5 < V_D < 0.9$ usual, $\boxed{\text{appox. } V_D \approx 0.7\text{ V usually. (Si)}}$

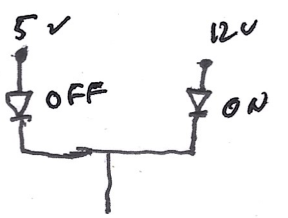
~~Diode w/ least resistance in parallel will be on.~~

$\boxed{V_{th} \approx V_{min},\ V_{th} = 0.7 \text{ approx}}$

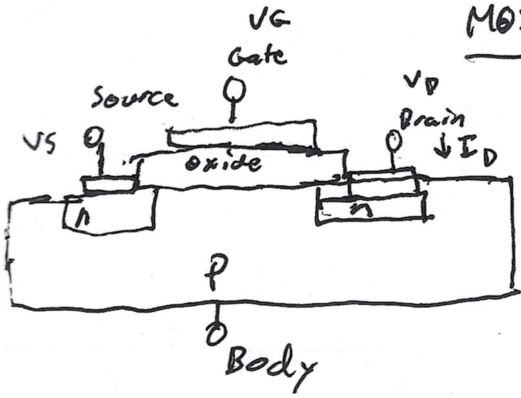
# How to Solve

1. Assume $V_D = 0.7$ (Si), $V_D = 0.3$ (Ge) (Ideal is 0, but not feasible)
2. Make assumptions abt forward/reverse bias
3. Solve the circuit w/ the assumptions.
4. Verify assumptions.

- Current will flow path of least Resistance / Higher Voltage wins
  - 

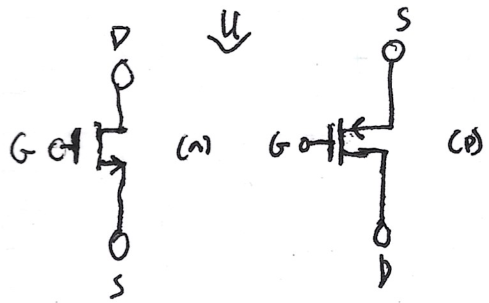
  - with resistors both can be on.
- 

# MOSFET Transistors




n-channel Mosfet
charge carrier = electrons

P-channel Mosfet switches p+n and uses holes as charge carrier making them hotter.




Current Goes D→S in n,
S→D in p.

$V_t$ = Threshold voltage for $V_G$
($I_D = 0$ if $V_G < V_t$) Off switch

about n type (p same but vs)
⇓

$V_{Dsat}$ = Saturation Voltage

$V_D << V_{Dsat}$ – Linear $I_D$ increase
$V_D < V_{Dsat}$ – Logarithmic $I_D$ increase till $V_{Dsat}$
$V_D > V_{Dsat}$ – constant $I_D$

$V_{Dsat} = V_G - V_t$
$V_{DS} = V_D - V_S$
$V_{GS} = V_G - V_S$

Transconductance parameter (based on construction)

$k = \mu_n C_{ox} \left(\frac{W}{L}\right)$

$$I_D = k\left((V_{GS} - V_t)V_{DS} - \tfrac{1}{2}V_{DS}^2\right) \quad V_{DS} \le V_{GS} - V_T,\ V_D \le V_G - V_T \quad \text{Triode region}$$

On Switch

$$I_D = \frac{k}{2}(V_{GS} - V_t)^2 \quad V_{DS} \ge V_{GS} - V_T,\ V_D \ge V_G - V_T \quad \text{Saturation region}$$

$\boxed{(V_G > V_t) \text{ for both}}$

Amplifier

Triode/Sat. Boundary is at $V_{DS} = V_{GS} - V_T$

$I_D = \frac{K}{2} V_{Dsat}^2$ @ transition.

$V_{GIA}$ – cutoff → sat
$V_{GIB}$ – sat → triode

~~$I_D$ ... for $V_G$~~

can be used for amps / logic gates. ⇒

Triode ⇒ ~~Logic On~~ Amp
Cutoff ⇒ Logic off
Saturation ⇒ ~~Amp~~ Logic ON

(controlled by Voltages)

$\boxed{I_G = 0,\ I_D = I_S}$

Most transistors specify k and $V_t$