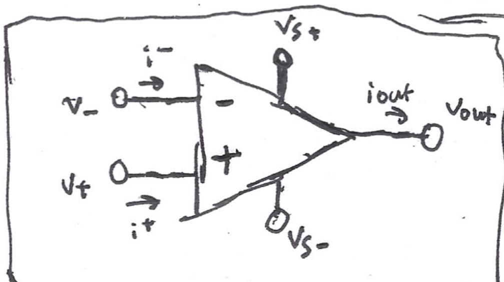
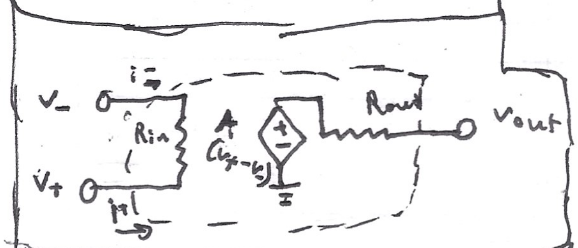
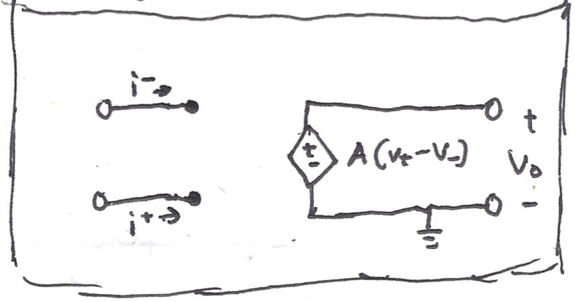
Ideal Op-Amp





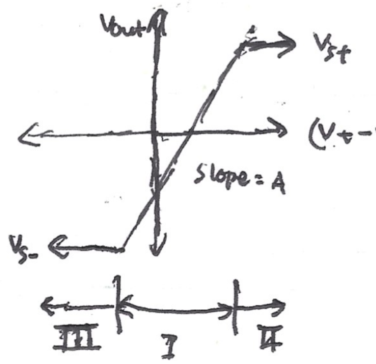


~~Ideal Model~~

Typical Parameters

$A = 10^4 \quad V_{s+/-} = 10V$

$(V_+ - V_-) = \pm 1mV$



To maintain linear region, we use negative feedback.

Ideal Op-Amps:

$R_i = \infty$

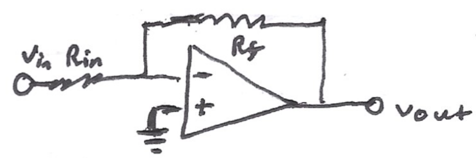
$A = \infty$

$R_o = 0$

$i_- = i_+ = 0$

$V_- = V_+$

Inverting Amp

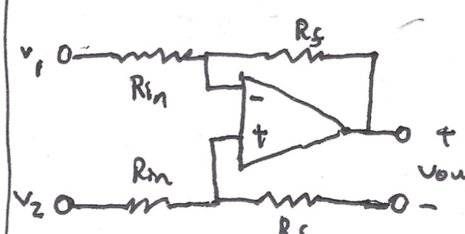


$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

Non Inverting Amp

$$V_{out} = \left(1 + \frac{R_F}{R_{IN}}\right) V_{in}$$

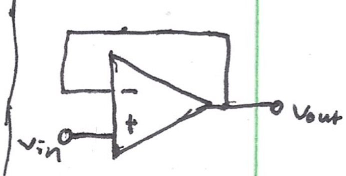
Differential Amp



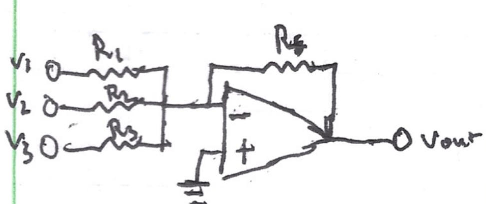
$$V_{out} = \frac{R_F}{R_{in}}(V_2 - V_1)$$

Voltage follower

(High Imp buffer)

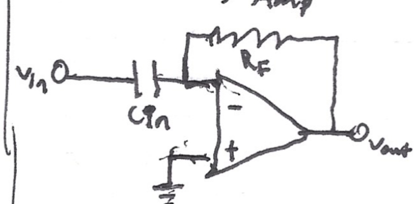


Summing Amp



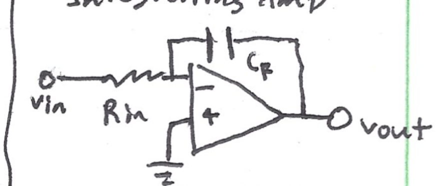
$$V_{out} = -R_f\left(\sum V/R_{in}\right)$$

Differentiating Amp



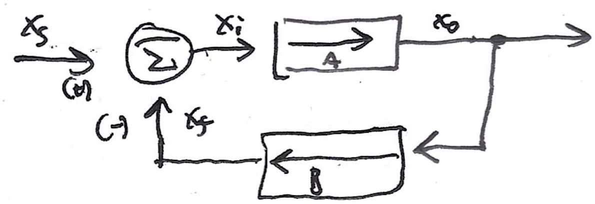
$$V_{out} = -R_F C_{in} \frac{dV_{in}}{dt}$$

Integrating Amp



$$V_{out} = -\frac{1}{R_{in}C_F}\int_0^t V_{in}(\lambda)\,d\lambda$$

Feedback Loops



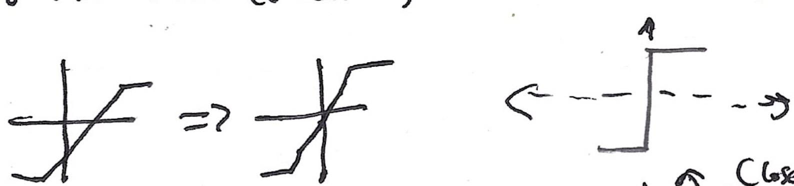
$x_o = A x_i$ $\quad x_f = \beta x_o$

A = open-loop gain $\quad \beta$ - feedback factor

$x_i = x_s - x_f \Rightarrow$ Closed-loop gain: $A_f = \frac{x_o}{x_s} = \frac{A}{1+A\beta}$

$A\beta >> 1 \Rightarrow A_f = \frac{1}{\beta}$

To use offset correction, a differential amp is used.



Closed ~~Open~~ Loop

Open Loop

$(v_+ - v_-) <<$
$A \approx (10^4)$
$R_o <<$
v in

Semi ideal
$(v_+ - v_-) <<$
$A \approx (10^4)$
$R_o = 0$
$i_- = i_+ = 0$

$v_+ - v_- = 0$
$A \to \infty$
$R_o = 0$
$i_- = i_+ = 0$

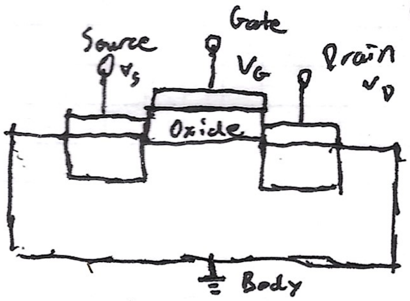
In freq. dep. analysis: replace everything with the impedance.

ECE 20002 Chpt 2

Field-Effect Transistors

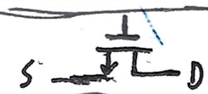
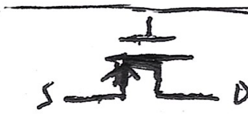
★ Review 2K1 Chpt 13 on basics over n-channel enhancement mode MOSFETs.

Creation of a MOSFET



The wells will be doped and the silicon substrate will be doped oppositely. Source is usually slightly more doped.

- n-doped = excess electrons a.k.a electrons = charge carriers (Usually Ph doped)
- p-doped – excess holes a.k.a holes = charge carriers (Usually B doped)
- Some mosfets will have a channel between the two wells with the same doping (depletion mode).
- A layer of oxide seperates the gate and substrate that is insulating, meaning $\boxed{I_G = 0}$ can be assumed.

Symbols	enhancement mode (OFF)	depletion mode (ON) ← (Normal State)
n-channel		
p-channel		

(Arrows indicate direction of conv. current flow).

Ideal MOSFET

V_{TH} = Threshold Voltage ⇒ n-ch. enh. + p-ch. dep. = $V_{TH} > 0$
n-ch. dep. + p-ch. enh. = $V_{TH} < 0$

To turn on enh. mode MOSFETs; Gate-Source must be polarized opposite to charge carriers ($+V_{GS}$ for n, $-V_{GS}$ for p).

To turn off dep. mode MOSFETs, Gate-Source must be polarized same to charge carriers ($-V_{GS}$ for n, $+V_{GS}$ for p).

(Most of the times body + source are @ same potential). tech. transistors are driven by volt. diff. between Gate + Body.

The Gate operates like a capacitor w/ capacitance rep. as C_{ox}.

~~All Chan. require~~

$$\boxed{\text{Cutoff} = \begin{array}{l} V_{GS} \le V_{th} \text{ for n-ch.} \\ V_{GS} > V_{th} \text{ for p-ch.} \end{array}}$$

ECE 20002 Chpt 2 cont'd

Ideal MOSFETs cont'd

Triode-Sat. Boundary: $\boxed{V_{DS(sat)} = V_{GS} - V_{Th} \quad \text{or} \quad V_{GD} = V_{th}}$

★ $V_{DS} < V_{DS(sat)}$ = Triode Region

- Complete channel connects Source + Drain
  - × Length - L, Cross sec. Area - A
- V_{DS} and I_D are linearly related, the ohmic/linear/triode region of a transistor.
  - × $R = \frac{\rho L}{A}$, ρ depends on channel and charge concentration

★ $V_{DS} \geq V_{DS(sat)}$ = Saturation Region

- Channel pulls back from drain making R ↑↑.
- V_{DS} will be high enough to allow charge to still flow
- I_D is <u>ideally</u> indep. from V_{DS}.

Ineq. flipped for p-channels. + Must not be in Cutoff for these regions to be reached.

k_n/k_p - fixed parameters of a transistor for n/p channel MOSFET.

$k_n = \mu C_{ox} \frac{W}{L}$ μ - charge mobility, W/L - width + length of channel

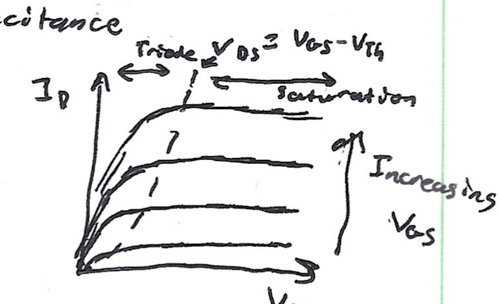
(units $\frac{mA}{V^2}$) (use Abs in eq) C_{ox} - oxide layer capacitance

These eqs are - when PMOS

$$I_{D(triode)} = k_n \left((V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

$$I_{D(saturation)} = k_n \frac{(V_{GS} - V_{th})^2}{2} = k_n \frac{V_{DS(sat)}^2}{2}$$

$$I_{D(cutoff)} = 0$$



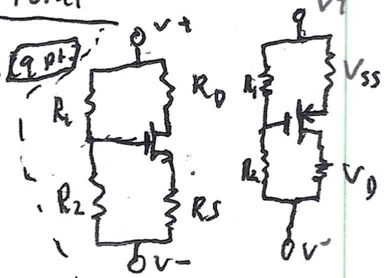
n-mosfet graphs

Solving

1. ID V_{GS} + V_{DS} (control Voltages)
2. Use KVL, writing ID as a func. of Control Voltages
3. Assume Saturation first.
4. Utilize $I_G = 0$ for KVL.

Designing Op. Point

To obtain operating pt. (Q pt.) of (V_{DS}, I_D) w/ 4 resistors →



We want this point to be at ($\frac{1}{2} V_{DS(max)}$, $\frac{1}{2} I_{D(max)}$) or the mid. of $\int$ n-ch p-ch the load line. Then Derive V_{GS} and use KVL to find V_G. $I_G = 0$ so use $R_1 + R_2$ for voltage divider to set V_G. $R_1 + R_2$ should both be big so low power is dissapated.

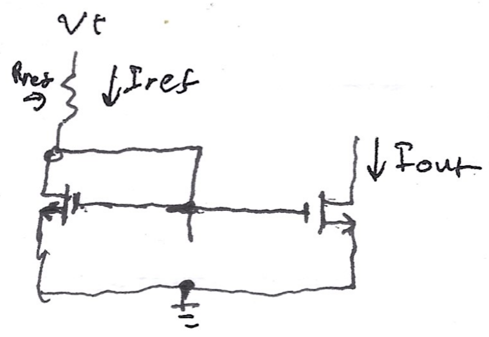
$$I_{D(max)} = \frac{V^+ - V^-}{R_D + R_{SS}}$$

$$V_{DS(max)} = V^+ - V^-$$

- A sho

- A: I=

Indep. Current Source



$M_1 = M_2$

$I_{out} = I_{ref}$

Changing R_{ref} will change I_{ref}/I_{out}

w/ diff channel parameters

$k_{n2} \neq k_{n1} \Rightarrow I_{out} = \alpha I_{REF}$

Channel Length Modulation

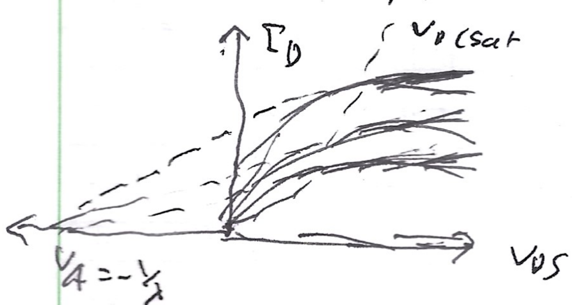
- Non-ideality of drain current: $I_{D(sat)}$ is not truly indep. of V_{DS}

As V_{DS} continues to increase past $V_{DS(sat)} = V_{GS} - V_{Th}$, the channel will draw back further from the drain.

$$\boxed{\begin{aligned} I_{D(sat)} &= \tfrac{1}{2} k_n (V_G - V_{Th})^2 (1 + \lambda(V_{DS} - V_{DS(sat)})) \\ &= \tfrac{1}{2} k_n V_{D(sat)}^2 (1 + \lambda(V_{DS} - V_{DS(sat)})) \end{aligned}}$$

where λ is the clm. parameter

$I_D - V_{DS}$ graph



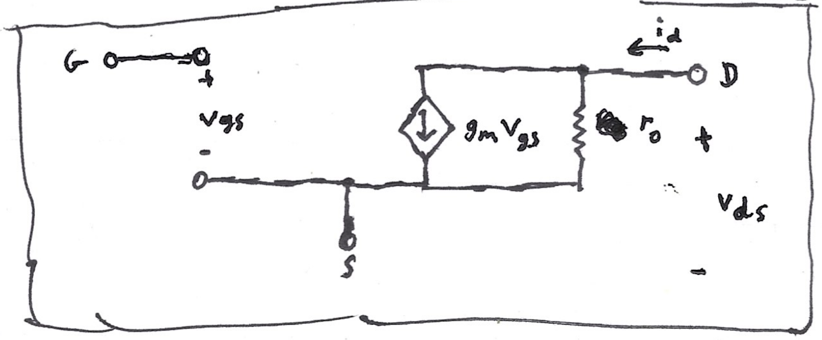
Extrapolating the saturation current lines backward should cause them to intersect @ $I_D = 0$,

$V = -\frac{1}{\lambda} = V_A$ (early Voltage)

This results in an output resistance we need to account for.

FET Small-Signal Model
- Mid Frequency

- This model is useful for most frequencies, a.k.a the mid-frequency model.
- It's useful for modeling MOSFETs with AC Voltage + Currents.
- AC model components are highly dependent on DC op. pt.
- we need to compute g_m and r_o to characterize this model and transistor amplifier circuits.



$r_o \approx \infty$ / open if no clm is present.

Transconductance - g_m

$$\boxed{g_m = \frac{i_d}{v_{gs}} = \frac{\partial I_D}{\partial V_{GS}}} \implies \boxed{g_m = k_n (V_{GS} - V_{Th}) = \sqrt{2 k_n I_D}}$$

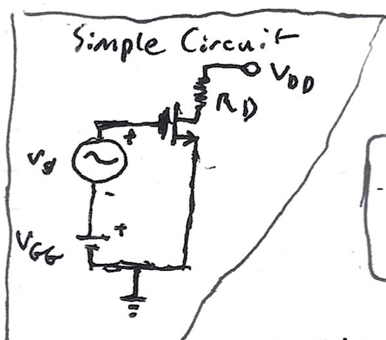
units: $1/\Omega$

~~{I_D = DC current}~~

This is derived assuming $v_{GS} = V_{GS} + v_{gs} \approx V_{GS}$
The same assumption is used for current.

i_d has a transconductance that varies with $\sqrt{I_D}$ - DC quiescent current.

Using the load line mentioned before, AC input will move back + forth from open-circuit voltage ($V_{oc} = V_{DD}$ here) and the short-circuit current (here, $I_{sc} = \frac{V_{DD}}{R_D}$).



~~I_D~~ i_d

$$\boxed{\begin{aligned} i_D &= \tfrac{1}{2} k_n (v_{GS} - V_{th})^2 \\ &= \tfrac{1}{2} k_n (V_{GS} - V_{th})^2 + k_n (V_{GS} - V_{th}) v_{gs} + \tfrac{1}{2} k_n v_{gs}^2 \\ &= I_D + i_d \end{aligned}}$$

$$\boxed{i_d = k_n (V_{GS} - V_{th}) v_{gs} + \tfrac{1}{2} k_n v_{gs}^2} \Leftarrow$$

$v_{gs} << V_{GS} - V_{Th}$ for small signal models + maintains linear model.
($|v_{gs}| < 0.2(V_{GS} - V_{th})$)

$$\implies \boxed{i_d \approx k_n (V_{GS} - V_{th}) v_{gs}}$$

(Clm) $\Rightarrow$ $(1 + \lambda(V_{DS} - V_{DS(sat)}) \mp \lambda (V_{DS} - V_{GS})$
Param for AC

ECE 20002 — Chpt 4 cont'd

Output Resistance - r_o

$$\boxed{r_o = \frac{\partial V_{DS}}{\partial I_D} \approx \frac{1}{\lambda I_D}}$$

Internal Output Res. ↓ w/ ↑I_D (DC curr)

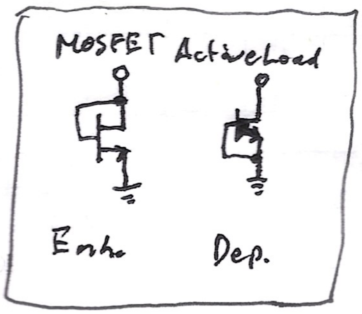
MOSFETs as Active Loads

Replacing a Resistor with a transistor to reduce DC voltage drop with a high small signal AC Impedence.

Slope of I-V = Inverse of AC resistance $\lambda \downarrow = r_o \uparrow$

Most MOSFETS r_o = 10-100s kΩ sometimes larger too.

This means small variations in output voltage shouldn't affect output current. Smaller I-V slope = larger v_{gs} swing allowed before large i_d. Large i_d increases chance that we move to edges of the load line, either ↑peak i_D = triode region or ↓ min i_D = cutoff region



⇐ This setup ensures MOSFETS don't reach cutoff.
This also means $I_D = 0 \Rightarrow V_{DS} = 0$
This implies $v_{gs} = 0$ at all times from the dep. mode model, meaning it will act like a resistor of r_o Ω in mid-freq. small signal models.

This is bc. the curr. source model will result in 0 curr. only leaving the resistor.

AC Analysis

1. Replace coupling + bypass capacitors w/ shorts.
2. All DC Sources = 0
3. Replace MOSFET w/ small signal equiv.

ECE 20002 | Chpt 5

Amplifier Circuits

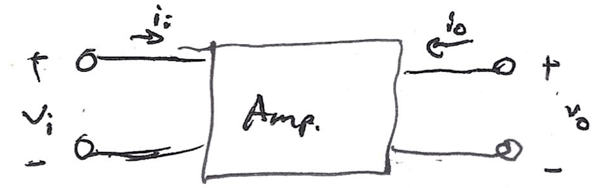
Black box interpretation:

$$A_{vi} = \frac{v_o}{v_i}$$

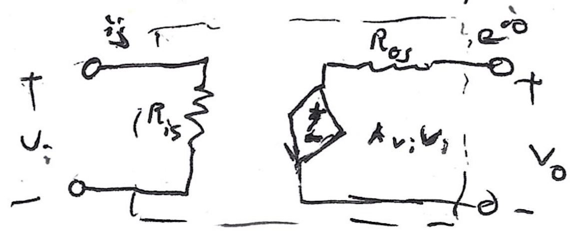
$$A_i = \frac{i_o}{i_i}$$

$$R_{is} = \frac{v_i}{i_i}\Big|_{i_o=0}$$

$$R_{os} = \frac{v_o}{i_o}\Big|_{v_i=0}$$



Dep Source Model



Intrinsic V gain of a MOSFET

$$\boxed{\mu_f = g_m r_o = \frac{1}{\lambda}\sqrt{\frac{2k_n}{I_{D(sat)}}}}$$

Common Source Amp

Simple CS Amp

To ensure Q-point stability we will use coupling capacitors (pass AC w/o changes DC bias) and bypass capacitors (for AC curr to get around resistors)

To solve for Amps.:

Seperate into DC + AC circuits.

DC Circuit:

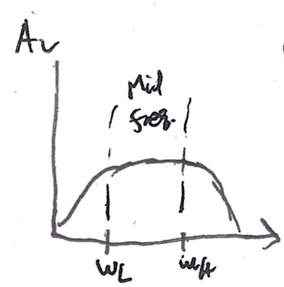
1. Capacitors = open
2. Inductors = Short
3. Deactivate AC Sources
4. Find Q-Point:

AC Circuit:

1. Capacitors = Short (@ op freq.)
2. Inductors = Open (@ op freq.)
3. DC V Sources = Deactivate / Short
4. DC I Sources = Deactivate / Open
5. Transistor ⇒ Small Signal Model
6. Analyze AC Response

When $A_v \ll \mu_f$, we can ignore r_o.

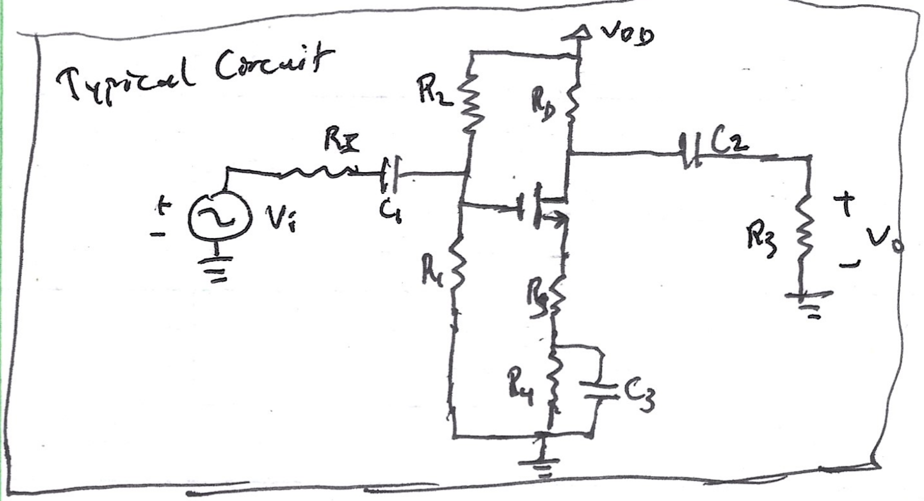
You can calc. A_v from $\frac{v_g}{v_i}$ and $\frac{v_o}{v_g}$



ω_L / ω_H = $1/\sqrt{2}$ of max gain

$\omega = 2\pi f$

Common Source Amp cont'd



Inverting Amp

$$\boxed{A_v = -\frac{g_m R_L}{1+g_m R_S}\left(\frac{R_G}{R_I+R_G}\right) \quad R_{in}=\infty,\ R_{out}=R_L}$$

where R_L = Eq Res from Drain to Ground in AC

and R_G = Eq Res from Gate to Ground in AC

To Calc R_C, use Req from AC model ~~Turnoff indep sources~~ (Turnoff indep sources + remove other Cs)

~~R_{C_1}~~

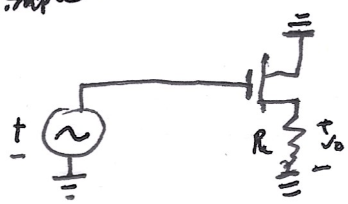
$$\boxed{\omega_{C_1} = \frac{1}{(R_I + R_1 \| R_2)C_1},\quad \omega_{C_2} = \frac{1}{(R_D + R_3)C_2},\quad \omega_{C_3} = \frac{1}{\left(\frac{1}{g_m} \| R_{SS}\right)C_3} \\ \omega_L = \max(\omega_{C_1}, \omega_{C_2}, \omega_{C_3})}$$

ω units - rad/s

ECE 20002 Chpt 6

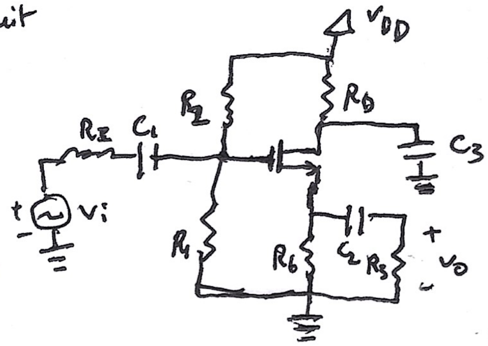
Common Drain Amp

Simple Circuit



Follower Circuit

Typical Circuit



$$A_v = \frac{g_m R_L}{1+g_m R_L}\left(\frac{R_G}{R_I+R_G}\right) \cong \frac{R_G}{R_I+R_G}$$

$R_{in} = \infty$

$R_{out} = 1/g_m$

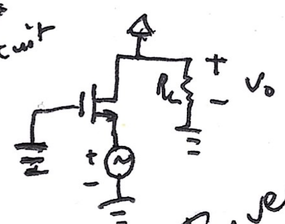
$$\omega_{C_1} = \frac{1}{R_I + R_1 \| R_2}, \quad \omega_{C_2} = \frac{1}{(R_3 + R_6 \| 1/g_m)\, C_2}$$

$\omega_L = \max(\omega_{C_1}, \omega_{C_2})$

(C_3 / R_D not always included)

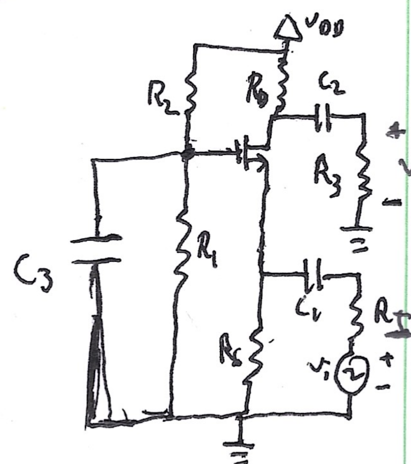
Common Gate Amp

Simple Circuit



Non Inverting Amp

Typical Circuit



$$A_v = \frac{g_m R_L}{1+g_m(R_I \| R_6)}\left(\frac{R_6}{R_I+R_6}\right)$$

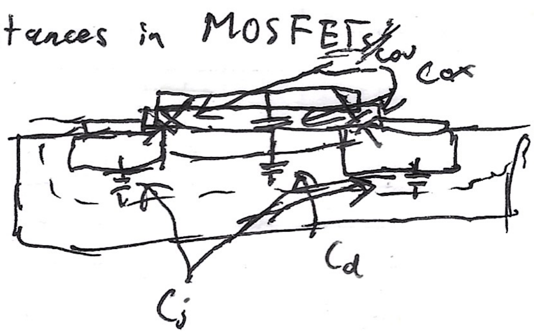
$R_{in} = 1/g_m$ $\quad R_{out} = R_L$

$$\omega_{C_1} = \frac{1}{(R_I + R_6 \| (1/g_m))\, C_1}, \quad \omega_{C_2} = \frac{1}{(R_D + R_3)\, C_2}, \quad \omega_{C_3} = \frac{1}{R_1 \| R_2\, C_3}$$

$\omega_L = \max(\omega_{C_1}, \omega_{C_2}, \omega_{C_3})$

High freq. cutoff

Capacitances in MOSFETs



Intrinsic Capacitances

C_{ox} - Oxide Capacitance

C_d - Depletion Capacitances

Parasitic Capacitances

C_j - Junction Capacitance

C_{ov} - Overlap Capacitance

$C_{ox} = C_{gs} + C_{gd}$

ω_H is defined by C_{gs} and C_{gd}.

Gate to Channel Capacitance:

$$C_{gs}(F) = WLC_{ox}$$

$$C_{gs} = \frac{2}{3}C_{gc} + C_{gso}X_{so}$$

$$C_{gd} = C_{gdo}X_{do}$$

C_{gso} - Cap of oxide & source overlap

C_{gdo} - Cap of oxide & drain overlap

X_{so} - len of oxide on source

X_{do} - len of oxide on drain

C_{gs} will connect Gate & Source in AC model.

$$\boxed{\omega_H = \frac{1}{R_{gs}C_{gs}}}$$

Max Linear Freq (Indep of System) Based on MOSFET

$$\boxed{f_T = \frac{1}{2\pi}\frac{g_m}{C_{gc}} = \frac{1}{2\pi}\frac{\mu}{L^2}(V_{gs}-V_T)}$$

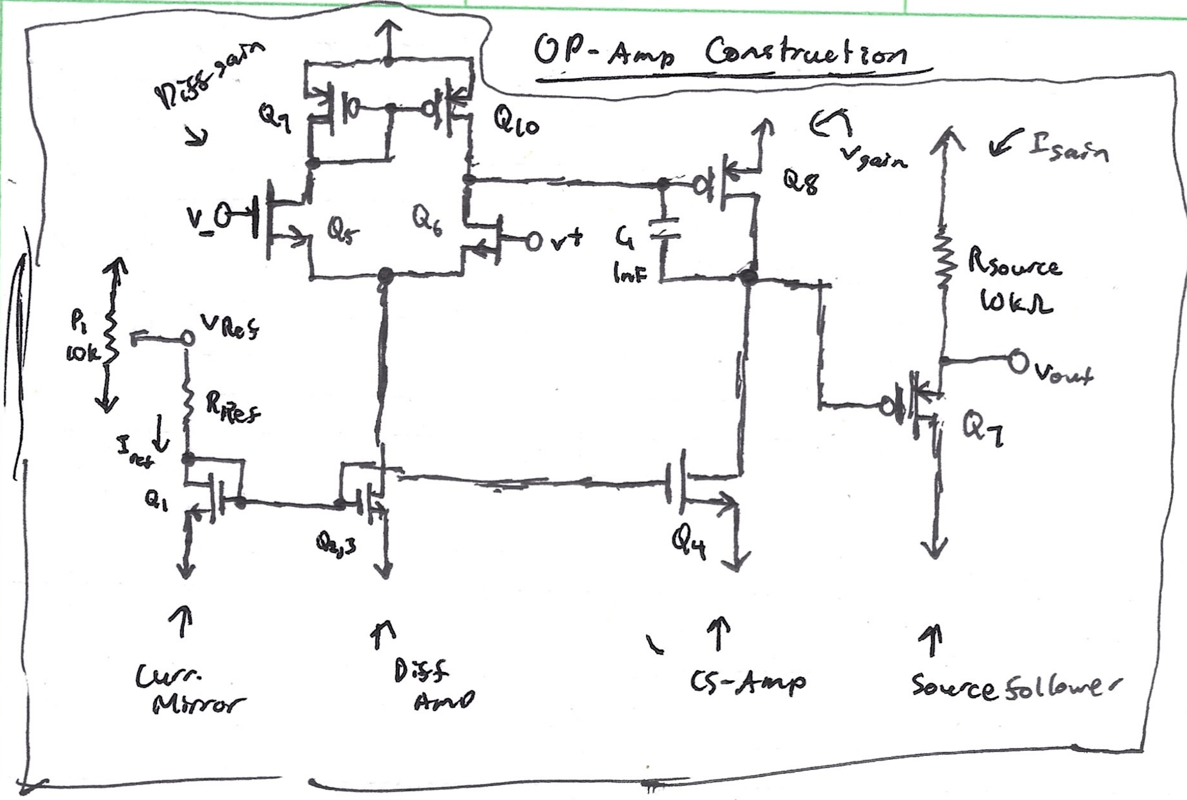
μ - Channel Mob. L - Channel Len.

C_{gs} can be replaced w/ any parallel capacitance to the curr. source.

aka capacitance connecting G-D, G-S, G-D

Sum $C_{gs}+C_{gd}$ in denom and take min vs C_{gs}/C_L freq.

OP-Amp Construction



Can Expand (

ODEs for RC + RL circuits

(Series) First order Circuits:

- RL Circuit: $V_L = L \frac{di_L(t)}{dt} \Rightarrow v_{in}(t) = L\frac{di_L(t)}{dt} + R i_L(t)$ (Derived from KVL)
- RC Circuit: $V_C = C\frac{dv_C(t)}{dt} \Rightarrow v_{in}(t) = RC\frac{dv_C(t)}{dt} + v_C(t)$
- Use KVL or KCL to determine relations and find homogenous + particular solutions, use conditions to solve for A & B + add them together. Then plus the variable value in the inductor/capacitor voltage equation.
- λ reps the natural frequencies.

ODEs for LC Circuits

Second Order circuits (Eulers eq: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$)

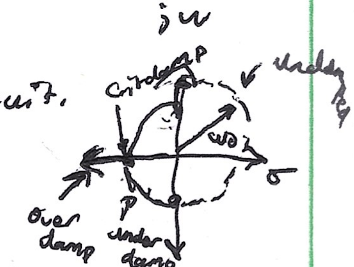
- LC Circuit: $V_L = L\frac{di_L(t)}{dt}$ $i_C = C\frac{dv_C(t)}{dt} \Rightarrow v_C(t) = LC\frac{d^2v_C(t)}{dt^2}$
- Use same strats as before

ODEs for RLC Circuits

- RLC Circuit: $v_{in}(t) = RC\frac{dv_C(t)}{dt} + LC\frac{d^2v_C(t)}{dt^2} + v_C(t)$

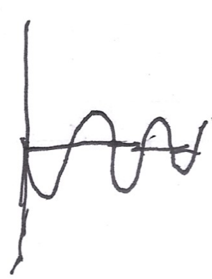
or

$$\frac{dv_{in}(t)}{dt} = R\frac{di_L(t)}{dt} + L\frac{d^2i_L(t)}{dt^2} + \frac{1}{C}i_L(t)$$



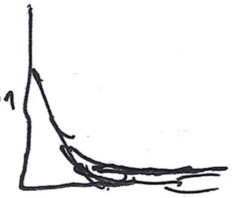
- Damping is done by adding resistors to an LC circuit.
- Over damping = Frequencies are real + distinct
- Critically damped = frequencies are real + equal
- Under damping = Frequencies are complex

Undamped: Under damped Critically damped Over damped



exp$(-\sigma_p t)$ where σ_p is the attenuation factor and ω is the damped res freq.

Particular gives center of oscillation



$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

$\omega_d = \sqrt{\omega_0^2 - \sigma_p^2}$

Root: $-\sigma_p \pm j\omega_d$

Switched Circuits

We know RL + RC circuits gen sol. $\Rightarrow x(t) = x(\infty) + [x(t_0^+) - x(\infty)]e^{\frac{-(t-t_0)}{\tau}}$

$\tau = RC$ or L/R where $x(t)$ can be i_L or V_C

given $x(t_1) = X_1$ $x(t_2) = X_2$ we can find the time between two states: $\boxed{t_2 - t_1 = \tau \ln\left[\frac{X_1 - x(\infty)}{X_2 - x(\infty)}\right]}$

- Steps:
  - Determine $t_1 \le t \le t_2$ (time range)
  - Find $x(0)$, $x(\infty)$, R_{TH}, τ, Voltages at t_1 + t_2.
  - Plug into solution, plug ~~solution~~ values into time equation to find $t_2 - t_1$!

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Function Transform

- Unit Step Function: $u(t) = \begin{cases} 0; & t<0 \\ 1: & t\geq 0 \end{cases}$
- Rectangular Function: $c(t) = \begin{cases} 0 & t<t_1 \\ 1 & t_1\leq t_1 < t_2 \\ 0 & t_2 \leq t \end{cases} = u(t-t_1) - u(t-t_2)$
- Unit Impulse Dirac Function: $\delta(x) = \begin{cases} \infty & x=0 \\ 0 & x\neq 0 \end{cases}$
  - $\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1$, $\frac{d}{dt} u(t) = \delta(t)$

Convolution

- ODE methods are tedious for large circuits, so use convolution
- Convolution relates inputs (excitations) to outputs (responses) using a circuit model operated on by the input function to get the out

$x(t)$ in $\rightarrow$ $\boxed{\text{Linear network}}$ $\rightarrow$ $y(t)$ out

- Convolution of $\boxed{f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau}$ or (for a time period) $= \int_0^t f(\tau)\, g(t-\tau)\, d\tau$ (shown below)
  - moving g reflected across y across the graph. area of intersection is $f(t) * g(t)$
- $\boxed{x(t) * \delta(t) = x(t)}$ => $x(t) * \delta(t) = \int_{t_0}^{t_0} x(\tau)\, \delta(t_0-\tau)\, d\tau = x(t_0)$

$x(t) * \delta(t) = x(t_0) * \int_{t_0-}^{t_0+} \delta(t_0-\tau)\, d\tau$

Use when function is shifted T $\rightarrow$ • Time shifting: $x(t) * \delta(t-T) = \int^{t-T} x(\tau)\, \delta((t-T)-\tau)\, d\tau = x(t-T)$

- $y(t) = x(t)u(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, \cancel{u(t-\tau)}\, d\tau$

$\boxed{x(t)\, u(t) * u(t) = \int_0^t x(\tau)\, d\tau\ u(t)}$ - converting time domain set it so unit step is where the top bound = bottom bound.

- Alg props:
  - $f(t) * g(t) = g(t) * f(t)$
  - $[f(t) * g(t)] * h(t) = f(t) * [g(t) * h(t)]$
  - $f(t) * [g(t) * h(t)] = f(t) * g(t) + f(t) * h(t)$

Time shifting Simple:
$x(t) * \delta(t-T) = x(t-T)$

Discrete Convolution

$u[n] = \begin{cases} 0 & n<0 \\ 1 & n\geq 0 \end{cases}$ $\delta[n] = \begin{cases} 0, & n\neq 0 \\ 1, & n=0 \end{cases}$ where $n \in \mathbb{Z}$

$$f[n] * g[n] = \sum_{m=-\infty}^{\infty} f[m]\, g[n-m]$$

- Uses same graphical shift as regular conv.
- Same alg. properties as reg. conv.

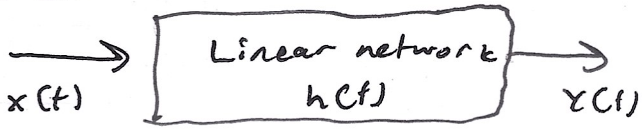
- Convert complex funcs into discrete

Max spacing = 0.2 (FWHM)
Preffered = 0.1 (FWHM)
where FWHM is width at 50% amp.
Preffered extension of peak tails = 20(FWHM)

↑ Accuracy with ↓ time intervals.

Impulse Response

- Linear Time Invariant Systems (LTIs)
  - produce output signals related to inputs linearly + time invariantly
  - Diagram:



or $\quad y(t) = x(t) * h(t)$

- Using $x(t) = \delta(t)$, we can find $h(t)$.
  - * $y(t) = \delta(t) * h(t) = h(t)$
  - * $h(t)$ is the Impulse Response

- The ODEs for RLC circuits give us the step response.
  - To find impulse response from step response, take the derivative of the step response.
  - $v_{in} = u(t)$ to find step response.
    or
    $i_{in} = u(t)$

Laplace Transformations

- Transforms from time domain to frequency domain (Diffeq → Alg)
  - Reciprocal of time is frequency. Can solve in freq and convert back.
- One sided Laplace Transform:

$$\boxed{\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt}$$

where s is the complex frequency. $s = \sigma + j\omega$

- Laplace Transforms of functions:

$$\boxed{\mathcal{L}\{\delta(t)\} = 1}, \quad \boxed{\mathcal{L}\{\delta(t-t_0)\} = e^{-s\{t_0\}}}$$

$$\boxed{\mathcal{L}\{u(t)\} = \frac{1}{s}} \leftarrow s > 0$$

- Properties

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2} f(t)\right\} = s^2F(s) - sf(0) - \frac{df(0)}{dt}$$

$$\mathcal{L}\left\{\int_{-\infty}^{t} f(\tau)\, d\tau\right\} = \frac{F(s)}{s} + \frac{\int_{-\infty}^{0} f(\tau)\, d\tau}{s}$$

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

$$\mathcal{L}\{f(t-t_0)\} = e^{-st_0} F(s) \leftarrow \text{Time shift}$$

$$\mathcal{L}\{e^{-at} f(t)\} = F(s+a) \leftarrow \text{Freq. shift}$$

$$\boxed{\mathcal{L}\{f(t) * g(t)\} = F(s)\, G(s)} \qquad \boxed{\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)}$$

- Use PFD to decompose fractions for inverse LT.
- Actual formula rarely used:

$$\frac{1}{j2\pi} \oint_{\Gamma} F(s)\, e^{st}\, ds$$

- Final Value Theorem

$$\lim_{t \to \infty} f(t) = f(\infty) = \lim_{s \to 0} sF(s)$$

Remember upper case is replaced

Current - Voltage Rels. in S-domain (freq. domain)

- Regular Response is applicable for any time
- Zero-State Response is when initial cond. are zero.

	Reg Resp.	Zero-State
Resistors		
$V_R(s)$	$R\,I_R(s)$	$R\,I_R(s)$
$I_R(s)$	$G\,V_R(s)$	$G\,V_R(s)$
Inductors		
$V_L(s)$	$L(s\,I_L(s) - i_L(0))$	$sL\,I_L(s)$
$I_L(s)$	$\frac{1}{L}\left(\frac{V_L(s)}{s} + \frac{\int_{-\infty}^{0} v_L(\tau)\,d\tau}{s}\right)$	$\frac{1}{sL}V_L(s)$
Capacitors		
$V_C(s)$	$\frac{1}{C}\left(\frac{I_C(s)}{s} + \frac{\int_{-\infty}^{0} i_C(\tau)\,d\tau}{s}\right)$	$\frac{1}{sC}I_C(s)$
$I_C(s)$	$C(s\,V_C(s) - v_C(0))$	$s(V_C(s))$

- Impedance can be defined as $\boxed{Z(s) = \frac{V(s)}{I(s)}}$. Admittance is $\boxed{Y(s) = \frac{1}{Z(s)}}$
- This means: and Admittance:

$$\boxed{\begin{aligned} Z_R &= R \\ Z_L &= sL \\ Z_C &= \frac{1}{sC} \end{aligned}} \qquad \boxed{\begin{aligned} Y_R &= G \\ Y_L &= \frac{1}{sL} \\ Y_C &= sC \end{aligned}}$$

- We can do this to solve odes. Use Impedences to find V/I in s-domain, and use inverse Laplace to get the time domain equations.

Initial Conditions

- Capacitor initial conditions.

$$I_C(s) = C(sV_C(s) - v_C(0^-))$$

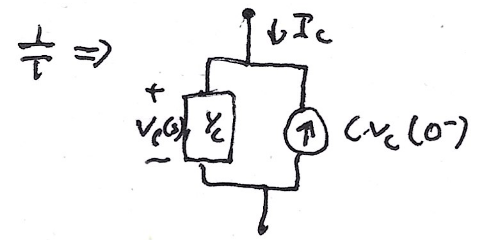
$$Y_C = sC$$

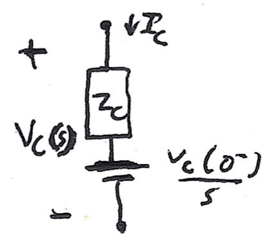
$$\Downarrow \quad I_C(s) = V_C(s)Y_C(s) - CV_C(0^-)$$

or

$$\boxed{V_C(s) = I_C(s)Z_C(s) + \frac{v_C(0^-)}{s}}$$

$$Z_C = \frac{1}{sC}$$





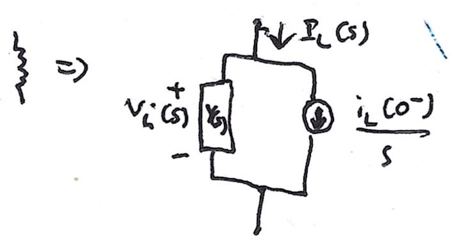
- Inductor initial conditions

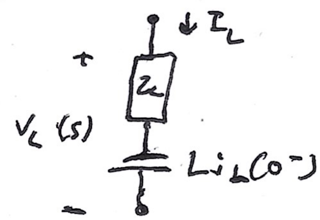
$$\boxed{I_L(s) = V_L(s)\,Y_L(s) + \frac{i_L(0^-)}{s}}$$

$$Y_L = \frac{1}{sL}$$

or

$$V_L(s) = I_L(s)\,Z_L(s) - Li_L(0^-)$$





- Use series ~~models~~ when circuit is series and parallel model when circuit is parallel.
- Use superposition to calculate voltages across internal impedences (calc with source and with init. cond. seperately). Then use initial condition to find properties across the component.
- Complete Response = Zero-Input Resp. (ZIR) + Zero-State Resp. (ZSR)
  - ZIR - Response to init conds. + turning off all inputs.
  - ZSR - Response to input sig. + turning off all init. conds.
  - Forced Resp. - Resp. from components not assoc. w/ natural freq (aka roots of chareq.)
  - Natural Resp. - Resp. from components assoc w/ natural freq
  - Transient Resp. - Resp. that changes w/ time (No sinusodial/constant)
  - Sinusodial Steady State Resp. - Resp. that is sinusodial w/ time.

Chpt 23 + 25

Transfer Function

- From $y(t) = h(t) * x(t) \Rightarrow Y(s) = H(s)X(s)$

  Out ↑ In ↑

- Depending on the question, voltage & current vars can be assigned to $Y(s)$ or $X(s)$. We find $H(s)$ by finding $Y(s)$ when $X(s) = 1$ and using $H(s) = \frac{Y(s)}{X(s)} \overset{X(s)=1}{\Rightarrow} H(s) = Y(s)$.
  - Initial conds = 0 (No stored energy)
  - No Indep. Sources.

The Complexity Plane + Stability

- Imped. + Admit. + Transfer functions are rational funcs:

$$H(s) = \frac{P(s)}{Q(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \dots + b_m s^m} \qquad (m \leq n)$$

$$\boxed{H(s) = K \frac{(s-z_1)^{q_1}(s-z_2)^{q_2}\dots(s-z_m)^{q_m}}{(s-p_1)^{r_1}(s-p_2)^{r_2}\dots(s-p_n)^{r_n}}}$$

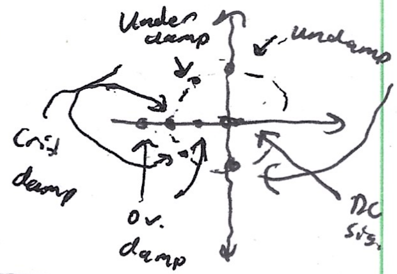
p_n are the poles of $H(s)$, r_n is order of repeated poles
z_n are the zeroes of $H(s)$, q_m is order of repeated zeroes
K is the gain constant

- Once $H(s)$ is found, response to any input can be calced (if the in var is the same)
- Trans. func. shows info of system through its form.
- Freq. Resp $s = j\omega$ or $s = \sigma + j\omega$ (remember $\omega_0 = \frac{1}{\sqrt{LC}}$ Radius, the damping circle.)
- X - pole, O - zeroes
- Poles show nat. response, $H(s)$ inverse laplace is nat. response
  - order of pole > 1, multiply term by t^n. ~~(sub jw and solve for real/imag)~~
- When doing inverse laplace, we can tell stability.
  - If order of pole > 1 ($At^{k-1}u(t)$), unstable, else ($Au(t)$) stable. ($\frac{A}{s}$ or $\frac{A}{s^k}$)
  - $p_i > 0$ = unstable, $p_i < 0$ = stable ($\frac{A}{s-p_i}$)
  - $p_i < 0$ = stable ($\frac{A}{(s-p_i)^k}$)
  - Marginally stable ($\frac{A(s)+B}{(s^2+\omega^2)}$)
  - $a > 0$ = stable, $a < 0$ = unstable ($\frac{A(s)+B}{(s+a)^2+\omega^2}$)

Circuit cond finite as $t \to \infty$

Chpt 23 + 25 cont'd

The Complexity Plane + Stability cont'd



- Bounded input, bounded output Stability (BIBO)
  - Sols have to be bounded for stability ← Max vals

Means
- In: $\int_{-\infty}^{\infty} |x(t)|\, dt < K_I < \infty$
- Out: $\int_{-\infty}^{\infty} |y(t)|\, dt < K_O < \infty$

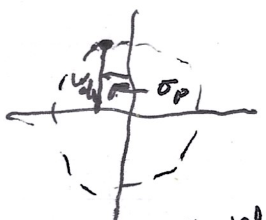
$\boxed{\text{ }}$

- Bibo is assured if all poles $\sigma_i < 0$ (Left) stability
- ~~Bibo~~ unstable if any pole $\sigma_i > 0$
- Order 1 poles + $\sigma_i = 0$ are marg. stable
- Order ≥ 1 poles + $\sigma_i = 0$ are unstable

- Zeroes show where the output will only be natural resp, not natural + forced.

Resonance

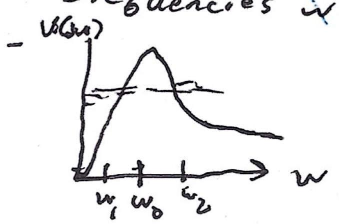
- RLC circuit is in resonance when the voltage and current are in phase at the input terminals.
  - σ_p is exp. attenuation factor (real part)
  - ω_d is damped res. freq. (im. part)
  - $\omega_0 = \sqrt{\omega_d^2 + \sigma_p^2} = \frac{1}{\sqrt{LC}}$



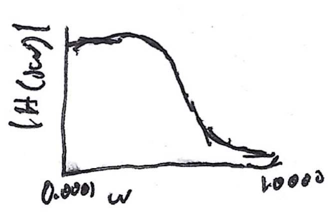
- Phasor method: replace $s = j\omega$
- ω_r is freq. when imp/adm. becomes purely real. Resonance frequency

Shorted → (L/C are off) – $\omega_r = \omega_0$ IN SIMPLE CIRCUITS

- Max Voltage is prop. to resistance, all curr. flows through resistor.
- Bandwidth ⇒ $B_\omega = \omega_2 - \omega_1$ where ω_2 and ω_1 are frequencies where volt = max $V \cdot \frac{1}{\sqrt{2}}$



- Phase angle: $\varphi = \tan^{-1}\frac{\omega}{\sigma}$ ← $\sigma + j\omega$
  - To convert an algebraic phasor into phasor form do this $\sigma + j\omega \Rightarrow \sqrt{\sigma^2 + \omega^2} \angle \tan^{-1}\left(\frac{\omega}{\sigma}\right)$
  - ex. $Z(j\omega) \Rightarrow |Z(j\omega)| \angle \tan^{-1}\left(\frac{\omega}{\sigma}\right) \Rightarrow |Z(j\omega)| \angle \varphi$
- We can graph changes in the phasor while changing ω, there are two graphs, $|Z(j\omega)|$ vs ω and φ vs ω which gets us phasor from frequency.
- Bode Plots approx. mag plots:
  - $\boxed{|H(j\omega)|_{dB} = 20 \log_{10} |H(j\omega)|}$
  - $\omega = 2$ rad/s is cutoff / half-power freq between low freq. + high freq.



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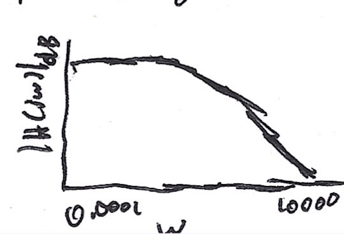
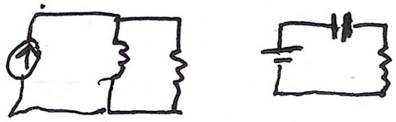
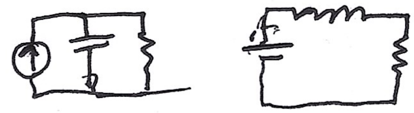


Filter Basics





High pass filters
- Allows ↑ freq.

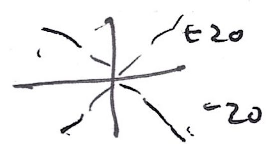
Low pass filters
- Allows ↓ freq.

- $|H(j\omega)|_{dB} > 0$ = Amplification, < 0 = Attenuation.
- $(s+z) \Rightarrow (s+\omega_p)$ where ω_p is ang. freq. at pole
  - Convention is $(s+z) \Rightarrow z(\frac{s}{z}+1) \Rightarrow z(\frac{s}{\omega_p}+1)$

Bode Plots

$|H(s)| = s = \omega \Rightarrow |H(j\omega)_{dB}| = 20\log(\omega)$

$|H(s)| = \frac{1}{s} = \frac{1}{\omega} \Rightarrow |H(j\omega)_{dB}| = -20\log(\omega)$ $\Rightarrow$

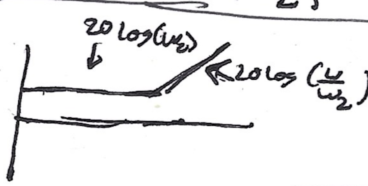


$H(s) = (s+z) \Rightarrow H(j\omega) = \omega_z(1+\frac{j\omega}{\omega_z})$

$\Rightarrow |H(j\omega)_{dB}| = 20\log(\omega_z) \quad \omega \to 0$

$|H(j\omega)|_{dB} = 20\log(\omega_z) + 20\log(\frac{\omega}{\omega_z}) \quad \omega \to \infty$

⇓



$H(s) = \frac{1}{s+p} \Rightarrow |H(j\omega)_{dB}| = -20\log(\omega_p) - 20\log\left[\sqrt{1^2+\frac{\omega^2}{\omega_p^2}}\right]$

$|H(j\omega)|_{dB} = -20\log(\omega_p) \quad \omega \to 0$

$|H(j\omega)|_{dB} = -20\log(\omega_p) - 20\log(\frac{\omega}{\omega_p}) \quad \omega \to \infty$

Bode plots

- $H(s) = K \dfrac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \Rightarrow H(j\omega) = K \dfrac{(j\omega-z_1)(j\omega-z_2)\cdots(j\omega-z_m)}{(j\omega-p_1)(j\omega-p_2)\cdots(j\omega-p_n)}$

$$\Downarrow$$

$$\boxed{|H(j\omega)| = K \frac{\prod_{k=1}^{m} |j\omega - z_k|}{\prod_{l=1}^{n} |j\omega - p_l|}, \quad \angle H(j\omega) = \sum_{k=1}^{m} \angle(j\omega - z_k) - \sum_{l=1}^{n} \angle(j\omega - p_l)}$$

Magnitude & Frequency Scaling

- Multiply Impedence by K_m (divide C by K_m, multiply R/L by K_m)
  - Includes curr. controlled sources
  - Divide by K_m for voltage controlled sources

← Mag Scaling

- This results in no change in zeroes or poles
  - $|H|$ could change.
- H has no change for dimensionless H (simple $\frac{Z_0}{Z_{in}}$)

- Resistors are not freq. scaled
- $L_{new} = \frac{L_{old}}{K_f}$
- $C_{new} = \frac{C_{old}}{K_f}$
- $H(s)_{new} = H\left(\frac{s}{K_f}\right)$
- $K_f > 1$ ↑ peak width, $K_f < 1$ ↓ peak width

↑ Freq. scaling.

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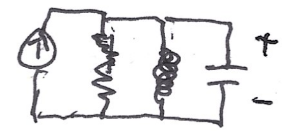
Bandpass Response



- Voltage Response Properties.
  - Max V is prop. to resistance
    - $Z_{in}(j\omega) = R$ @ $\omega = \omega_r$
  - All current flows through R @ $\omega = \omega_r$
  - Width of voltage response depends on R, L, C

↑ = narrow
↓ = wide

Sharpness of peak →

  - Quality factor ⇒ $\boxed{Q = \frac{\omega_m}{B_w} = \frac{\omega_r}{B_w}}$
  - Bandwidth $B_w = \omega_H - \omega_L$

- H_m - max of transfer func, ω_m - when $|H(j\omega)| = H_m$

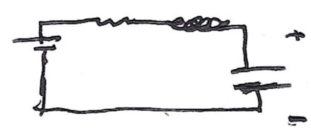
$$\boxed{\begin{aligned} \omega_H &= \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} + \frac{1}{2RC} \\ \omega_L &= \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} - \frac{1}{2RC} \end{aligned} \approx \begin{aligned} \omega_H &= \omega_m + \frac{B_w}{2} \\ \omega_L &= \omega_m - \frac{B_w}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} B_w &\approx \omega_H - \omega_L = \frac{1}{RC} \\ Q &= \frac{\omega_m}{B_w} = \omega_0 RC \end{aligned}} \quad \boxed{H_m = R}$$

← specific to parallel

$$\boxed{\begin{aligned} B_w &= \frac{R}{L} \\ Q &= \omega_0 \frac{L}{R} \end{aligned}}$$

← specific to series



$$\boxed{H_m = 1/R}$$

$$\boxed{\begin{aligned} \omega_H &= \omega_m + \frac{B_w}{2} \\ \omega_L &= \omega_m - \frac{B_w}{2} \end{aligned}}$$

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Passive Low Pass Filters

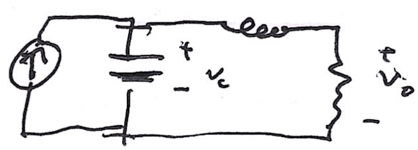
- Higher order = faster cutoff.
- First order:

$$\boxed{H(j\omega) = \frac{1}{1+j\omega}}$$

- Second order

$$\boxed{H(s) = Z_{in}(s)}$$

w_z - corner freq of the zero.



- Filter characterization
  - Analysis
    - × Define $H(s)$
    - × Convert to SSS $s = j\omega \Rightarrow H(j\omega)$
    - × Construct Mag. Eq. $|H(j\omega)|$
  - Steps
    - × Specify type: (high, low, band)-pass
    - × Construct transfer func.
    - × Design Circuit.

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Butterworth LPF

- Transfer function

$$\boxed{|H(j\omega)| = \frac{|K|}{\sqrt{1+(\frac{\omega}{\omega_c})^{2n}}}}$$

$\omega_c = \omega_{3dB}$ - corner freq

n = filter order

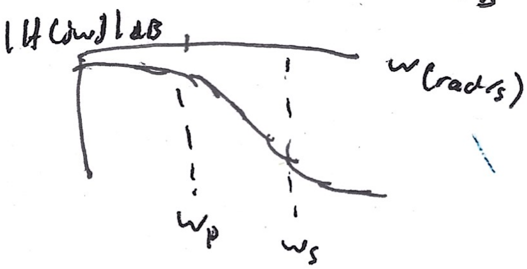
K = constant

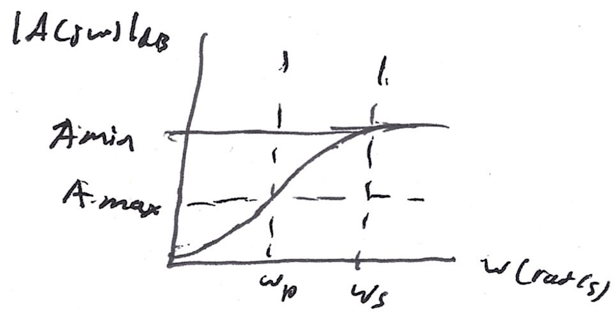
- Normalized B LPF Transfer functions. ($K=1, \omega_c = 1$ rad/s)
  - $n=1$ $H(s) = \frac{1}{s+1}$
  - $n=2$ $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$
  - $n=3$ $H(s) = \frac{1}{s^3+2s^2+2s+1}$

- $|A(j\omega)|_{dB} = |H(j\omega)|_{dB}^{-1}$
- $A(j\omega)_{dB} = -H(j\omega)_{dB}$

ω_p - pass band

ω_s - stop band





- Filter Design Steps. Design Specifications: $\boxed{\{A_{max}, A_{min}, \omega_p, \omega_s\}}$
  - Find filter order - $(\frac{\omega_s}{\omega_c})^{2n} \geq 10^{0.1A_{min}} - 1$
  - Select transfer function (use normalized)
  - select circuit structure
  - Find + adjust ω_c if $A_{max} \neq 3dB$ ($\omega_c = \omega_p$ if $A_{max} = 3dB$)
  - Magnitude + freq. scale as needed

1. $$\boxed{n_{min} \geq \frac{\log\left(\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}\right)}{2\log\left(\frac{\omega_s}{\omega_p}\right)}}$$

   (select first int min)

2. Choose transfer from normalized options
3. If source R_s is known, add 1Ω in series w/ filter and use mag scaling
   If load R_L is known, add 1Ω in parallel w/ filter
4. ω_c is half power freq, $\boxed{\frac{\omega_p}{(10^{0.1A_{max}}-1)^{\frac{1}{2n}}} \leq \omega_c \leq \frac{\omega_s}{(10^{0.1A_{min}}-1)^{\frac{1}{2n}}}}$

Butterworth HPF

- HPF Design
  - Convert HPF to LPF
  - Design Butterworth LPF
  - Convert LPF to HPF
    - x C to L and L to C
    - x freq + mag scaling.

1. $\omega < \omega_p \quad A < A_{max}$ $\quad\Rightarrow\quad$ $\omega < \omega_s \quad A > A_{min}$

   $\omega > \omega_s \quad A < A_{min}$ $\qquad\quad$ $\omega > \omega_p \quad A < A_{max}$

$$\boxed{\Omega = \frac{\omega_p}{\omega} \; , \; \Omega_p = \frac{\omega_p}{\omega_p} = 1 \; ; \; \Omega_s = \frac{\omega_p}{\omega_s} > 1}$$

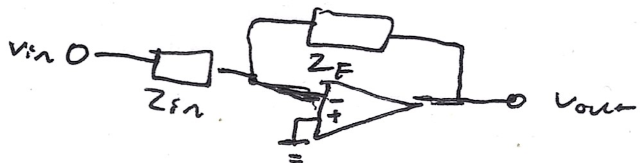
2. Replace all ω_s w/ Ω_s.

3. $$\boxed{\begin{array}{ll} C_{LPF} \rightarrow L_{HPF} & = \dfrac{1}{C_{LPF}} \\ L_{LPF} \rightarrow C_{HPF} & = \dfrac{1}{L_{LPF}} \end{array}}$$

Active LPF

- Passive -
  - Requires Inductor (> 2nd order) which is most non-ideal element
  - Passband gain < 0dB (other than peaking)
- Active
  - Can avoid inductors
  - Pass band gain > 0dB

- Op-Amp LPFs



$$\Rightarrow \boxed{H(s) = \frac{V_{out}}{V_{in}} = -\frac{Z_F(s)}{Z_{in}(s)}}$$