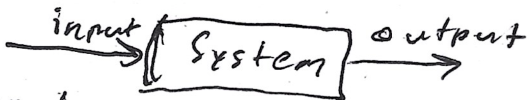
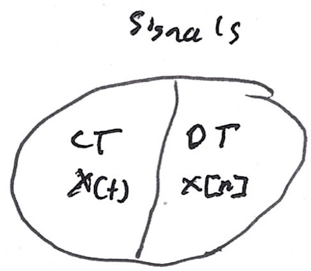
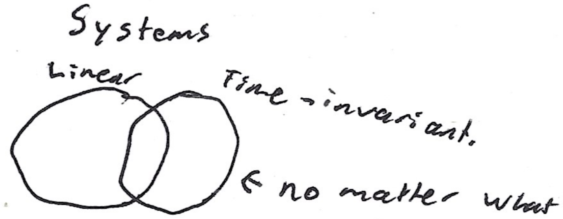
Week 1

- Signals
  - Something that contains info.
  - A series of data where $t \in \mathbb{R}$ (continuous (CT)) or $t \in \mathbb{N}$ (discrete (DT))
- Systems
  - Anything that takes signals as inputs and outputs.



  - Can have CT in/out and/or DT in/out



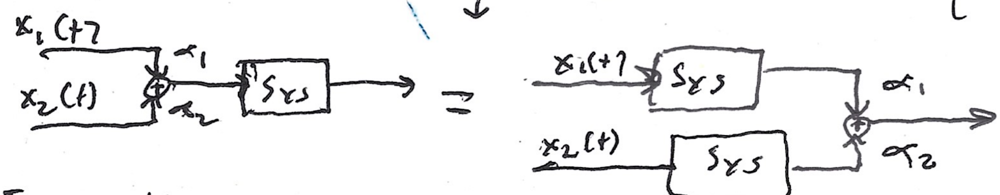


← no matter what time input is given, resp. is the same.

↑ $y(t-T) = x(t-T) * h(t)$

(More info in week 5)

- Linear Systems
  - Is linear iff both are equal



- DT signals are only valued on integer time values
- CT signals are valued for all real vals. for time
- Energy
  - CT signals → $\boxed{E = \int_{t_1}^{t_2} |x(t)|^2 \, dt}$
  - DT signals → $\boxed{E = \sum_{n=n_1}^{n_2} |x[n]|^2}$
  - evaluate at $-\infty$ and ∞ for total energy
- Power
  - Avg Power: CT: $P = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} |x(t)|^2 \, dt$ DT: $P = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[t]|^2$
  - Total Power: CT: $P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$ DT: $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[t]|^2$
  - Inst. Power: $|x(t)|^2$ or $|x[t]|^2$
- No finite energy + ∞ power signals.

Week 2

- Time-index transformations (All are linear trans.)
  - Time-shift - $y(t) = x(t-t_0)$
    - × $t_0 > 0$ is right shift
    - × $t_0 < 0$ is left shift
  - Time-reversal - $y(t) = x(-t)$: flip across y-axis
  - Time-scaling - $y(t) = x(\alpha t)$: where $\alpha > 0$
    - × $\alpha > 1$ is squishing the graph
    - × $\alpha < 1$ is stretching the graph.
- Classifications of signals
  - CT vs DT
  - ∞ / finite × energy / power - 4 types
  - Even/odd
  - Periodic: $x(t)$ is periodic with period T if
    $x(t) = x(t-T)$ or $x[n] = x[n-T]$
- Fund. Period is the smallest period of a function
  - If $x(t) = x_{Re}(t) + x_{Im}(t)j$ is periodic, $x_{Re}(t)$ and $x_{Im}(t)$ are periodic.
- Period
  - How to det:
    - × Inspection
    - × If x_1 and x_2 are periodic then $x_1(t) + x_2(t)$ has period of the lcm of the two periods (no lcm = not periodic)
- Even/odd
  - Even if symmettric across y axis, or $x(t) = x(-t)$
  - Odd if symmetric across $y=x$ or $x(t) = -x(-t)$
  - Can be neither.
- Complex exponential signals: $x(t) = Ce^{\alpha t}$ where C/α are complex.
  - $x(t) = |C| e^{\sigma t} e^{j(\omega t + \phi)}$ - $|r(t)| e^{\angle x(t) j}$
  - × C Scales, $\sigma = 0$, $\sigma > 0$, $\sigma < 0$

  - × Term 3 is periodic.
- $x(t) = x_{even}(t) + x_{odd}(t)$ for any x
  - $x_{even}(t) = \frac{x(t) + x(-t)}{2}$, $x_{odd}(t) = \frac{x(t) - x(-t)}{2}$
  - $$\boxed{x(t) = \underbrace{\frac{x(t) + x(-t)}{2}}_{\text{even}} + \underbrace{\frac{x(t) - x(-t)}{2}}_{\text{odd}}}$$

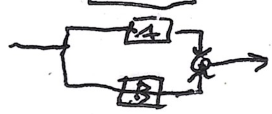
Week 3

- CT harmonically related signals (HrCEs): $e^{jk\omega t}$
  complex exponentials
  - k is an integer, are periodic
  - fund period $= \frac{2\pi}{|k|\omega}$
- Similar for DT signals, just splitted discretely.

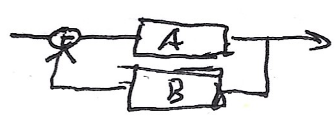
Week 4

- $u(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t \geq 0 \end{cases}$ + $\delta(t) = \begin{cases} 0 & \text{if } t \neq 0 \\ \infty & \text{if } t=0 \end{cases}$
- $\cancel{\int \delta(t)}$
  - $\delta(t) = \frac{d}{dt} u(t)$
  - $u(t) = \int_{-\infty}^{t} \delta(s)\, ds$
  - $u(t) = \int_{0}^{\infty} \delta(t-s)\, ds$
  - $\int_{-\infty}^{\infty} \delta(t) = 1$
  - $\delta[n] = u[n] - u[n-1]$
- $x(t) = \int_{-\infty}^{\infty} x(s)\, \delta(t-s)\, ds = x(t) * \delta(t)$
- Concatenation - can be enveloped by a larger black box.
  - Serial:

  - Parallel:



  - Mixed
  - Feedback:



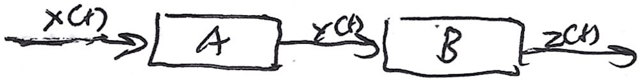
- Classifications:

Systems:
- Memory
- Invertibility
- Causality
- Stability
- Time-Invariance
- Linearity

Signals:
- DT vs CT
- periodic vs ~~non~~ aperiodic
- ∞ / finite — energy / power
- even/odd/neither

week 4 cont'd

- With Memory vs memoryless
  - memoryless is if $y(t)$ doesn't matter on $x(s)$ for $s \neq t$
- Invertible vs non-invertible
  - Invertible is:



$x(t) = z(t)$.

* Then B is the inverse of A and A is invertible.

week 5

- LTI systems are easy to analyze and lotta systems can be approximated as a LTI.
  - Linear (have explained earlier)
  - Time-Invariant = Doesn't matter when we input vals.
- Use δ to find $h(t)$.
- Review 2k2 convolution.
- LTI system props.
  - Comm., Distr., Assoc.
    - Comm.: conv. w/ $h(t)$ or $x(t)$
    - Distr.: conv w/ $h_1(t) + h_2(t)$ or indv and add.
  - Impulse resp is $h(t)$
- LTI classifications
  - Memoryless if $\boxed{h(t) = k\,\delta(t)}$ or $\boxed{\begin{array}{l} h(t) = 0 \text{ for } n \neq 0 \\ h[n] = 0 \text{ for } t \neq 0 \end{array}}$
  - Causality (dep on past + pres.) if $\boxed{h(t) = 0 \text{ for } t < 0}$, $h(t) \neq 0$ only for $t \geq 0$
  - Invertibility if h_{inv} exists so $\boxed{h(t) * h_{inv}(t) = \delta(t)}$
  - Stability if $\boxed{\int_{-\infty}^{\infty} |h(t)|\,dt < \infty \text{ or } \sum_{-\infty}^{\infty} |h[n]| < \infty}$

ECE 301 | Week 6 + 7

week 6

- LTI system input w/ C.E.
  - $\boxed{y(t) = e^{j\omega t} H(j\omega) \Rightarrow H(j\omega) = \int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds}$

    ~ coeff.

  - freq: $\omega_0 = \frac{2\pi}{T}$

- Fourier series representation: writing $\boxed{x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}}$ — FS synth eq.

  a_k — coeff, $e^{jk\omega_0 t}$ — HRCE test signal

  - a_0 - DC comp, $a_{-1} e^{j(-1)\omega_0 t}$ - 1st order Harm. Comp.
  - $\boxed{a_k = \int_T x(t) e^{-jk\omega_0 t} dt \text{ (integrating across one period)}}$ — FS analysis eq.

    ex. $\frac{1}{T}\int_0^T = \int_T$

  - Can calc. a_k by inspection or direct computation
    - × Inspection:
    - × Direct Computation: Calc a_0 and general a_k.

Week 7

- Fourier series exist for continuous funcs. and cont. funcs w/ holes where cont.

- $x(t) \xleftrightarrow{FS} (a_k, \omega_0)$

  time domain — freq-domain

- F.S. Properties
  - Linearity - $\boxed{c_k = Aa_k + Bb_k}$, ω_0 ← $Ax(t) + By(t)$ given ω_0 is the same
  - Time Shift - $\boxed{b_k = a_k e^{-jk\omega_0 t_0}, \omega_0}$ ← $x(t - t_0)$
  - Time Reversal - $\boxed{b_k = a_{-k}, \omega_0}$
  - Time Scaling - $\boxed{b_k = a_k, \alpha\omega_0}$ ← $x(\alpha t)$
  - Differentiation - $\boxed{b_k = (jk\omega_0) a_k, \omega_0}$ ← $\frac{d}{dt} x(t)$
  - Multiplication - $\boxed{c_k = a_k * b_k, \omega_0}$ ← $x(t) \cdot y(t)$ given same ω_0
  - Parseval's Relationship - $\boxed{\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{-\infty}^{\infty} |a_k|^2}$
    - Power consumption law.

    $x(t) \longleftrightarrow a_k$

Week 8

- DT Fourier Series. (Period N)
  - Synthesis formula: $\boxed{x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}}$
  - Analysis formula: $\boxed{a_k = \frac{1}{N} \sum_{n \in \langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}}$
- DT FS properties
  - Linearity, Time-shift, Time-Reversal are the same $\omega_0 \Rightarrow \frac{2\pi}{N}$
  - Difference - $\boxed{b_k = a_k(1 - e^{-jk\frac{2\pi}{N}}),\ \frac{2\pi}{N}}$ ← $x[n] - x[n-1]$
  - Parseval is the same.
- Convolution properties. (can convolve then FS or FS then convolve)
  - CTFS: $b_k = a_k H(jk\omega_0)$
  - DTFS: $b_k = a_k H(e^{jk\omega_0})$

Week 9

- CT Fourier Transform.
  - Synthesis formula: $\boxed{x(t) = \int_{\omega=-\infty}^{\infty} a_\omega e^{j\omega t} d\omega}$
  - Analysis formula: $\boxed{a_\omega = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt}$

$\Downarrow$

- $X(j\omega) = a_\omega \cdot 2\pi$, $X(j\omega) = \mathcal{F}(x(t))$, $x(t) = \mathcal{F}^{-1}(X(j\omega))$
- $x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$ ← Inv. FT
- $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ ← FT of $x(t)$

$$\boxed{\begin{tikzcd} x(t) \arrow[rr, leftrightarrow, "* h(t)"] \arrow[d, leftrightarrow, "FS"'] & & y(t) \arrow[d, leftrightarrow, "FS"] \\ a_k, \omega_0 \arrow[rr, leftrightarrow, "H(j\omega)"'] & & b_k = a_k \cdot H(jk\omega_0) \end{tikzcd}}$$

FT

Periodic ←

- $H(j\omega)$ is freq. response.

ECE 301 | Week 9+10

Week 9 contd

- $\text{sinc}(\theta) = \dfrac{\sin(\pi\theta)}{\pi\theta}$
- For general periodic $x(t)$ find FT
  - Find CTFS (w_0, a_k)
  - $\boxed{X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta(w - kw_0)}$
- CTFT properties
  - Linearity: $a x(t) + b y(t) \rightarrow a X(jw) + b Y(jw)$
  - Time-shift: $x(t-t_0) \rightarrow X(jw) e^{-jwt_0}$
  - Freq-shift: $x(t) e^{jw_0 t} \rightarrow X(j(w-w_0))$
  - Time Reversal: $x(-t) \rightarrow X(-jw)$
  - Time Scaling: $x(at) \rightarrow \frac{1}{a} X(\frac{jw}{a})$
  - Differentiation: $\frac{d}{dt} x(t) \rightarrow jw X(jw)$
  - Parseval's Relationship: $\int_{-\infty}^{\infty} |x(t)|^2 dt \rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$
  - Convolution: $x(t) * y(t) \Rightarrow X(jw) Y(jw)$
  - Multiplication: $x(t) y(t) \rightarrow \frac{1}{2\pi} X(jw) * Y(jw)$

$$
\begin{tikzcd}
x(t) \arrow[rr, leftrightarrow, "*h(t)"] \arrow[d, leftrightarrow, "FT"'] & & y(t) \arrow[d, leftrightarrow, "FT"] \\
X(jw) \arrow[rr, leftrightarrow, "H(jw)"'] \arrow[urr, phantom, "\updownarrow FT"] & & Y(jw)
\end{tikzcd}
$$

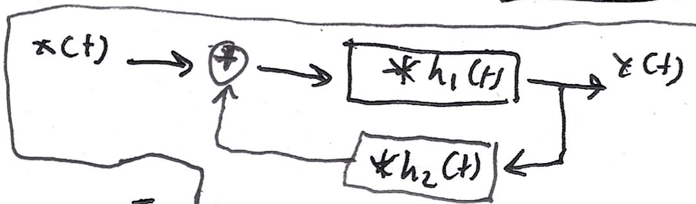
← aperiodic $\int x(js)\, Y(j(w-s))\, ds$ (works for periodic too)

Week 10

- Another way to find $h(t)$:
  - $\boxed{H(jw) = \dfrac{Y(jw)}{X(jw)}, \quad h(t) = \mathcal{F}^{-1}(H(jw))}$
  - Helps since its hard to gen. an impulse (∞ amplitude)
- Inverting Impulse resp
  - $\boxed{H_{INV}(jw) = \dfrac{1}{H(jw)}, \quad h_{inv}(t) = \mathcal{F}^{-1}(H_{INV}(jw))}$
  - $h(t) * h_{INV}(t) = \delta(t)$

ECE 301 — Week 10 + 11

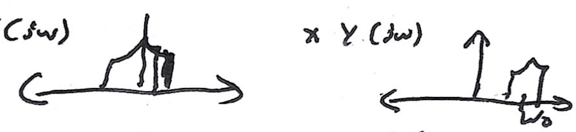
Week 10 cont'd

- 

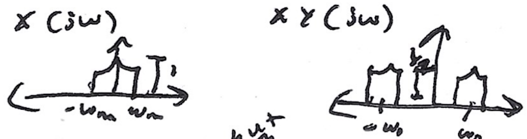
  - $Y(j\omega) = \dfrac{H_1(j\omega)}{1 - H_1(j\omega)H_2(j\omega)} X(j\omega)$
- Low pass filter:
  - $\boxed{h(t) = \dfrac{\sin(Wt)}{\pi t},\quad H_{LPF}(j\omega) = \begin{cases}1 & \text{if } |\omega| < W \\ 0 & \text{otherwise}\end{cases}}$
- Modulation: $y(t) = e^{j2\pi f_k t} \cdot x(t)$ (shifts to $2\pi f$)
  - ↑ freq for better transmission.
  - Prob is it has img. parts: $y(t) = \cos(f 2\pi t)\, x(t)$
    - * Splits signal in ½ w/ ½ Amp. @ $\pm 2\pi f$
    - * Amplitude Modulation (AM)
      - 500k → 1.6 MHz for AM radio

Week 11

- Amplitude Modulation: $y(t) = x(t)\, c(t)$ → carrier
  - $y(t)$ → modulated; $x(t)$ → modulating signal
  - Type 1: $c(t) = e^{j\omega_0 t}$
    - * $X(j\omega)$ * $Y(j\omega)$



    - * Disads: not sym, or real (can't transmit)
    - * Demod: $x(t) = y(t)e^{-j\omega_0 t}$
  - Type 2: $c(t) = \cos(\omega_c t)$ + LPF w/ ω_m cutoff
    - * $X(j\omega)$ * $Y(j\omega)$



    - * LPF ↓ qual but removes overlap.
    - * ω_c is chosen, ω_m is for no overlap, $\omega_r \geq \omega_m$ (by a little) ← recieving cutoff freq.
- How to send multiple signals
  - $x_1 \to$ LPF $\to \otimes \cos(\omega_a t)$
  - $x_2 \to$ LPF $\to \otimes \cos(\omega_b t)$ $\to \oplus \to y(t)$
  - $x_3 \to$ LPF $\to \otimes \cos(\omega_c t)$
  - $\omega_b - \omega_a > 2\omega_m$
  - $\omega_c - \omega_b > 2\omega_m$
  - usually $2\omega_m(1 + 10\%)$ ← guard band
  - Demod: $y(t) \to \boxed{BPF} \to \otimes \cos(\omega_a t) \to \boxed{LPF} \to \otimes 2 \to$
- Asynch Demod. of AM signals require Envelop detector.
- Frequency Division Multiplexing (FDM)

ECE 301 | Week 11+12

Week 11 cont'd

- AM single side band
  - Just take half of signal (its symmetric)

$x_1 \to LPF \to \otimes_{\cos(w_a t)}$ $\to$ $\oplus \to$

$x_2 \to LPF \to \otimes_{\cos(w_b t)}$ $\to$ $\oplus$

$Y \to BPF \to \otimes_{\cos(w_a t)} \to LPF \to \otimes_{4} \to x(t)$

Week 12

- DTFT (aperiodic $x[n]$)
  - $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw}) e^{jwn}\, dw$
  - $X(e^{jw}) = \sum x[n] e^{-jwn}$
    - $X(e^{jw}) \leftarrow DT$, $X(jw) \leftarrow CT$

      Periodic respect to w, 2π period
  - $X(e^{jw}) = 2\pi\,\delta(w - \pi/3)$ for $e^{j\frac{\pi}{3}n}$

  $$\boxed{X(e^{jw}) = 2\pi\,\delta(w - w_0) \text{ for } e^{jw_0 n}}$$

- DTFT properties
  - Linearity: $ax[n] + by[n] \leftrightarrow aX(e^{jw}) + bY(e^{jw})$
  - Time-Shift: $y[n] = x[n-n_0] \leftrightarrow Y(e^{jw}) = e^{-jwn_0} X(e^{jw})$
  - Freq.-Shift: $y[n] = e^{jw_0 n} x[n] \leftrightarrow Y(e^{jw}) = X(e^{j(w-w_0)})$
  - Time-Reversal: $y[n] = x[-n] \leftrightarrow Y(e^{jw}) = X(e^{-jw})$
  - Difference in time: $y[n] = x[n] - x[n-1] \leftrightarrow Y(e^{jw}) = (1 - e^{-jw}) X(e^{jw})$
  - Differentiation in freq: $n\,x[n] \leftrightarrow j\frac{d}{dw} X(e^{jw})$
  - Parseval's relationship: $\sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{jw})|^2\, dw$
  - Convolution Property: $x[n] * h[n] \leftrightarrow X(e^{jw}) H(e^{jw})$
  - Multiplication Property: $x[n]\,y[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})\, Y(e^{j(w-\theta)})\, d\theta$

~~DTFT pairs~~

- Duality - CT = cont., DT = disc., FS = disc., flip CT per., FT = cont, flip DT per.

  - CT FS

time	cont.	periodic
	↕	↕
freq	disc.	aperiodic

  - DTFS

time	disc.	periodic
	↕	↕
freq.	disc.	periodic

  - CT FT

time	cont.	aperiodic
	↕	↕
freq	cont.	aperiodic

  - DTFT

time	disc	aperiodic
	↕	↕
freq	cont.	periodic

ECE 301 — Week 13

Week 13

- Implementing a CT impulse response
  - Can use capacitors, resistors, etc. but u can only approximate
  - Better sol., sampling a CT ⇒ $x[n] = x(nT)$
- Sampling

$$x(t) \rightarrow \text{Sampling} \xrightarrow{x[n]} * h[n] \xrightarrow{Y[n]} \text{reconstruction} \rightarrow Y(t)$$

  - Sampling is $x[n] = x(nT)$ where T is the sampling period
  - We can sample and reconstruct:
    - ~~By~~ By just connecting the dots (when samp freq ↓ we lose too much data)
    - Or holding a data pt till next sample (" ")
      * easiest + good for large $\frac{2\pi}{T}$
    - Best Method: Impulse-train sampling (ITS)
  - Possible to reconstruct band limited signal perfectly
  - ITS:

$$x(t) \rightarrow x[n], T$$

$$x(t) \rightarrow \boxed{x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)} \quad \Big(x(t)\cdot \underbrace{\sum_{n=-\infty}^{\infty} \delta(t-nT)}_{p(t)}\Big)$$

  - good for conceptual analysis, not applicable
  - $\boxed{X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \;,\; \omega_s = \frac{2\pi}{T}}$ (ω_s is sampling freq.)

- To reconstruct pass through LPF w/ correct cutoff freq and multiply by T
  - Sampling theorem
    * $x(t)$ is band-limited, $|X(j\omega)| = 0$ if $|\omega| > \omega_m$ then perfect reconstruction is possible if

$$\boxed{\omega_s > 2\omega_m}$$

- Unideal ITS:

$$x(t) \rightarrow x[n],\ T \xrightarrow{x_p(t)} \hat{x}(t) \quad , \quad \boxed{\hat{x}(t) = T\,x_p(t) * \frac{\sin(\frac{\omega_s}{2}t)}{\pi t}}$$

$\omega_{cutoff} = \frac{\omega_s}{2}$ for Low pass

$$\boxed{\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(nT)\left(\frac{\sin(\frac{\pi}{T}(t-kT))}{\frac{\pi}{T}(t-kT)}\right)}$$

- Zero on hold + Others
  - $\hat{X}_{ZOH}(j\omega) = X_p(j\omega)\,H_0(j\omega)$ where $H_0(j\omega) = \underbrace{e^{-j\omega\frac{T}{2}}}_{\text{Timeshift}}\frac{2\sin(\frac{\omega T}{2})}{\omega}$
  - $\hat{X}_{LIN}(j\omega) = X_p(j\omega)\,H_1(j\omega)$ where $H_1(j\omega) = \frac{1}{T}\left(\frac{2\sin(\frac{\omega T}{2})}{\omega}\right)^2$
- Aliasing (overlapping freq.)
  - $\omega_0 \rightarrow \omega_s - \omega_0$ for $\omega_s < 2\omega_0$
    $\phi \rightarrow -\phi$

$$\cos(\omega_0 t + \phi) \;\Downarrow\; \cos((\omega_s - \omega_0)t - \phi)$$

Week 14

- CTFT to DTFT
  - $\boxed{X_d(e^{j\omega}) = X_p(j\frac{\omega}{T})}$

    $T = 2\pi$ $\quad$ $T \hat{=} \omega_s$
- Alternatives to Fourier transform
  - Some signals can't use FT
  - CT: Laplace Transform
  - DT: Z Transform
- Z transform
  - $\boxed{X(z) = \sum_{-\infty}^{\infty} x[n] z^{-n}, \quad z = \gamma e^{j\omega}}$, γ is exponential weighting
  - $X(z)$ only exists in the Region of Convergence (when the series converges)
  - just a DTFT but $\gamma \cdot e^{j\omega}$
  - $\boxed{x[n] = \gamma^n \mathcal{F}^{-1}(X(\gamma e^{j\omega}))}$
    - find γ such that $|z| = \gamma$ circle is in the ROC.
    - Use $X(z)$ for $Y(e^{j\omega}) = X(\gamma e^{j\omega})$
    - $x[n] = \gamma^n y[n]$
- Alternate ways of calc. z transform
  - Inspection - $X(z)$ given in terms of z
    - can be w/ power series
  - $y[n] = x[n] * h[n] \Rightarrow Y(z) = X(z) H(z)$